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Simulation of Project Completion Time with Burr XII Activity Distribution

Enobong Francis Udoumoh[1*], Daniel W. Ebong[2] and Iberedem A. Iwok[2]

[1]Department of Mathematics, Statistics and Computer Science, University of Agriculture, Makurdi, Nigeria.
[2]Department of Mathematics and Statistics, University of Port Harcourt, Port Harcourt, Nigeria.

Authors' contributions

This work was carried out in collaboration between all authors. Author EFU designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author DWE managed the analyses of the study. Author IAI managed the literature searches. All authors read and approved the final manuscript.

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Abstract

Project Evaluation and Review Technique (PERT) in conjunction with Monte Carlo simulation is no doubt a useful and an acceptable tool for the analysis of project networks. However, a common challenge in practice is the inability to confidently select appropriate input distribution to represent activity durations owing to scarcity of project network data. A pilot survey of bore hole drilling projects in Benue state, Nigeria revealed that most activity duration data do not follow the Beta distribution and some other commonly used activity duration distributions. Hence, the Burr XII distribution is introduced as input activity duration distribution in the simulation of stochastic project network. Some desirable properties that qualify the use of Burr XII distribution are also presented. Results obtained using the Burr XII activity duration distribution shows advantage over the traditional PERT-Beta.

Keywords: Project completion time; simulation; Burr XII distribution.

**Corresponding author: E-mail: udoumoh.francis@uam.edu.ng, uenobong@gmail.com;*

1 Introduction

The Critical Path Method (CPM) in conjunction with Project Evaluation and Review Technique (PERT) is a popular tool used by managers and management practitioners in project management. The main aim of CPM/PERT analysis is to obtain the distribution of project completion time. PERT was introduced to add some probability flavour to already existing critical path method (CPM).

The developers of PERT [1] assumed that activity times follow the generalized beta distribution with density

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{(x-a)^{\alpha-1}(b-x)^{\beta-1}}{(b-a)^{\alpha+\beta-1}} \; ; \; a < x < b, \alpha, \beta > 0$$

Where α and β are the shape parameters, $\Gamma(.)$ is the gamma function. The mean, variance and skewness are given as

$$\mu_x = a + (b-a)\frac{\alpha}{\alpha+\beta}, \; \sigma_x^2 = (b-a)^2 \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \text{ and } \gamma_1 = \frac{2(\beta-\alpha)\sqrt{\alpha+\beta+1}}{(\alpha+\beta+2)\sqrt{\alpha\beta}} \text{ respectively.}$$

However, given the judgemental activity time estimates: a-optimistic, m-mostlikely, b-pessimistic; the mean and variance were rather estimated to be $\hat{\mu} = (a + 4m + b)/6$

and $\hat{\sigma}^2 = (a + b)^2/36$ respectively. Hence, the method is commonly known as PERT-Beta approach.

There is no doubt that PERT-Beta provides useful estimates. Nevertheless, some of its assumptions introduce some potential sources of bias which led to the underestimation of project completion time. Some sources of bias as discussed in literature include: the seemingly intuitive adoption of beta distribution for activity duration, the approximate method of computing activity mean and variance, the assumption of independence among activities, the concentration on only the critical activities and ignoring the near critical activities, and the approximation of the distribution of project completion time by the normal distribution [2-5]. Some modifications have been done on the methods of estimating PERT-Beta activity parameters [6-8]. These modifications have not actually addressed the distribution and estimation based errors inherent in PERT-Beta approach [9].

The introduction of Monte-Carlo simulation in project network analysis has however helped relax some of the assumptions made in PERT-Beta procedure [10]. Moreover, results obtained via Monte Carlo simulation are almost the same as the ones obtained using exact procedures [3]. Hence, the Project Management Body of Knowledge [11] has adopted simulation as a reliable tool for the analysis of project networks. One of the steps to carrying out Monte Carlo simulation on project network is to define activity duration distribution. To this end, researchers have suggested many input distributions for activity duration. AbouRizk and Halpin [12] suggested the use of generalized Beta distribution and the Pearson System based on robustness with respect to their coverage on the skewness-kurtosis plane. Other distributions like; triangular [13], weibull [14], beta-rectangular [15], lognormal [16], normal [17], uniform [18], Lognormal distribution with Parkinson effect [19], Pearson III distribution [20] have also been used as input distributions for activity duration. However, choosing an input distribution that can support the use of Monte-Carlo simulation depend on basically two conditions; simplicity of the cumulative distribution function and also the ability of the distribution to 'realistically' model the underlying activity duration. Moreover, in PERT analysis, the robustness of the shape of the input distribution is also of great importance. Most of the probability distributions suggested in literature do not satisfy most of these conditions. For instance, the generalized beta distribution is quite robust to model varying activity shapes but its distribution form is not simple, hence, the reason for the PERT-Beta approximation [2,21].

In a different line of argument, Kamburowski [22] and Hajdu and Bokor [23] supported the PERT-Beta approximations. They said that there is no significant difference in PERT activity estimates using different input distributions. But Hajdu and Bokor [23], in a case study, considered the following judgemental estimates: $60-$ optimistic, $100-$ most likely, and $150-$ pessimistic whose skewness coefficient is only 0.09826. The beta distribution defined by this coefficient of skewness is the one that approximate the normal distribution which is also supported by PERT-Beta approximation [5,7]. Hence, there is no doubt about their conclusions. In spite of this, the graphical results that compared the PERT-Beta, triangular and uniform distributions still showed some obvious variations. It is possible that a set of data with much longer tail would have yielded worse results. Recently, Hahn and López Martín [24] proposed the use of tilted beta distribution to support the possible overstretched tail of some activity duration. On the whole, the popular belief is that most activity durations are positively skewed [6,16,25].

In this paper, Burr XII distribution is used as an input distribution for activity duration to estimate the project completion time. The rest of the paper is presented as follows: In section 2, a review of Burr XII distribution is presented, with highlight of its desirable properties that qualifies it as input activity duration distribution. Section 3 describes the pilot case study that led to the choice of Burr XII distribution. In section 4, we describe the process of the simulation of project completion duration, and finally a conclusion is drawn in section 5.

2 The Burr XII Distribution

Burr [26] formulated the Burr system distributions which comprise of twelve continuous parametric distributions. Among these is one of the most popular known as Burr XII distribution.

A random variable X is said to follow a 3 parameter Burr XII distribution with shape parameters c, k, and a scale α, if the probability density function is given as;

$$f(x) = ck\frac{x^{c-1}}{\alpha^c}\left(1+(\frac{x}{\alpha})^c\right)^{-(k+1)} ;\ x \geq 0, c > 0, \alpha > 0, k > 0$$

The cumulative distribution function is

$$F(x) = 1-(1+(\frac{x}{\alpha})^c)^{-k}$$

The r^{th} moment about the origin is given as

$$E(X^r) = \frac{k\alpha^r\Gamma(k-{}^r/_c)\Gamma({}^r/_c+1)}{\Gamma(k+1)} \qquad ; ck > r$$

Hence, the mean of X is

$$E(X) = \frac{k\alpha\Gamma(k-{}^1/_c)\Gamma({}^1/_c+1)}{\Gamma(k+1)};\ ck > 1$$

Variance of X is defined as

$$V(X) = E(X^2) - [E(X)]^2$$

$$\mathrm{V(X)} = \frac{k\alpha^2\Gamma\left(k-{}^2/_c\right)\Gamma({}^2/_c+1)}{\Gamma(k+1)} - \left(\frac{k\alpha\Gamma\left(k-{}^1/_c\right)\Gamma({}^1/_c+1)}{\Gamma(k+1)}\right)^2$$

Burr XII is a unimodal distribution at $X = [(c-1)/(ck+1)]^{\frac{1}{c}}$ if $c > 1$ and L-shape otherwise. Figs. 1 and 2 present the varying shapes of Burr XII probability density function with α =1.

The Burr XII distribution is a flexible distribution that can express a wide range of distribution shapes. For instance, if a random variable X has a Burr XII distribution, then $1/X$ has a Burr (III) distribution which is sometimes referred to as the Dagum distribution. Some popular distributions like the Exponential, Weibull and Pareto are special limiting cases of the Burr distribution, Rodriguez [27].

If c equals 1, Burr XII distribution becomes the log-logistic distribution. Hence, Burr XII is often termed the generalized log-logistic distribution. The log-logistic distribution is an important distribution that is used to model event history data and has similar shape with lognormal distribution which has been used for the analysis of activity times [16,19] Elsewhere, it was demonstrated that Burr XII distribution can model a lognormal random variable [28]. It has been demonstrated that Burr XII can fit a wide range of empirical data because it possesses a broad range of skewness and kurtosis. Burr XII region covers sections corresponding to the Pearson Type I, IV, and VI on the skewness-Kurtosis plane. Some sections of the normal, logistic, exponential, lognormal and extreme value distributions are also covered [27,29]. Some mixture densities like Weibull-Gamma and Weibull-Exponential and can be obtained from Burr XII distribution [30]. Summary of some special cases of a 2 parameter Burr XII distribution is presented in Table 1.

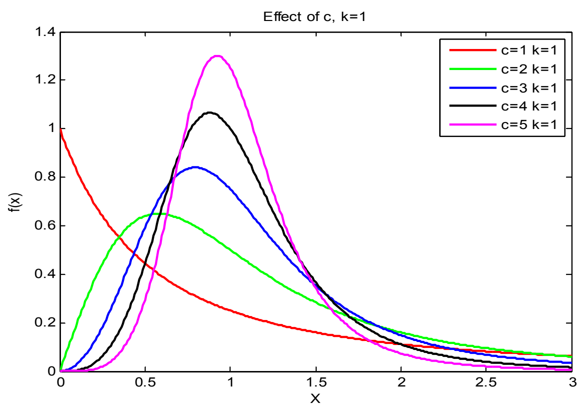

Fig. 1. Plots showing the effect of c when $k = 1, \alpha = 1$

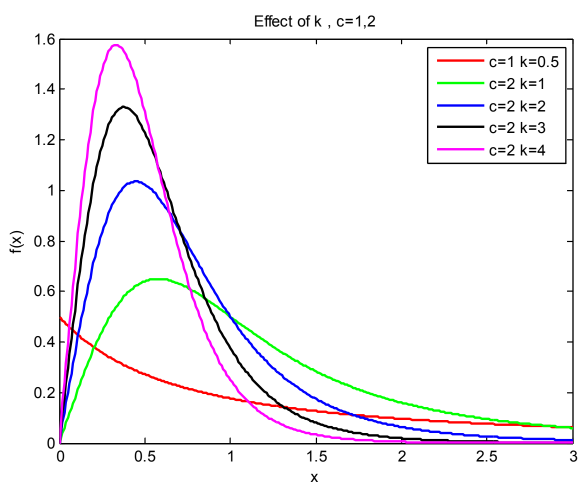

Fig. 2. Plots showing the effect of k when $= 1, 2$, $\alpha = 1$

Table 1. Special cases of 2P Burr XII distribution [28]

c	k	Distribution
c	∞	Weibull
1	∞	Exponential
∞	k	Generalized logistic
∞	1	Logistic
4.874	6.158	Approximate normal
∞	∞	Gompertz
1	1	Pareto

3 The Case Study

We conducted a study of water bore hole drilling projects in Makurdi, North Central region of Nigeria by Nile Drill Tech. limited. The essence of the study was basically to obtain information on the nature of activity time distributions. A total of 20 drilling project sites were observed. With the assistance of the site geologist the project was divided into 9 activities, namely; geographical survey, data analysis, punching, casing, gravel packing, connection of pipes to pump, connection of cables to pump, lowering of pump, and plumbing. For any borehole drilled, all of the 9 activities listed in Table 1 were carried out. All 20 bore holes were drilled at 100 meters depth and air drilling method was used for punching. We were able by observation and with the assistance of experts on site, to obtain information on the duration for each of the 9 activities across the 20 sites. Table 2 presents the descriptive statistics of these activities.

Table 2. Descriptive statistics of the duration of activities

Activity	Description	Mean	Median	Mode	Variance	Skewness	Min	Max
A	Site Survey	32.95	31	30	22.3658	1.4976	28	45
B	Analysis of data	63.1	61	60	34.7263	1.7203	56	80
C	Drilling	315.55	305.5	300	811.8394	0.9950	280	380
D	Casing	30.75	30	30	3.25	1.1940	28	35
E	Gravel Packing	16.55	16	15	6.7868	1.6038	14	24
F	Connection of pipes to pump	202.25	185	180	1782.8289	1.3595	150	300
G	Connection of cables to pump	23.95	24	25	7.8395	1.0320	20	32
H	Lowering of pump	3	3	3	0.6316	0.6991	2	5
I	Plumbing	103.7	100	100	113.8	1.0126	90	130

From Table 2, it could be observed that the coefficients of skewness for activities A to I are positive ranging from 0.6991 to 1.7203, supporting the general supposition that activity durations are often positively skewed [25]. A goodness-of-fit test was further carried out to select the best distribution for each activity. A combination of the kolmogorov Smirnov test, Anderson Darling test, and Chi-square tests were conducted to select the best fit distribution for the sample data. The analysis was done with the help of Easyfit 5.6 statistical software. Figs. 3-11 present the histograms of the best fit distributions of the 9 activities on bore hole drilling project. The following distributions made the best fit; 4-parameter Burr XII , 4-parameter Burr XII, 3-parameter Burr XII, Log-logistic (3P), Fretchet (3P), Log-logistic(3P), Fretchet (3P), Nakagami and General Extreme Value for activities A, B, ⋯ , I respectively.

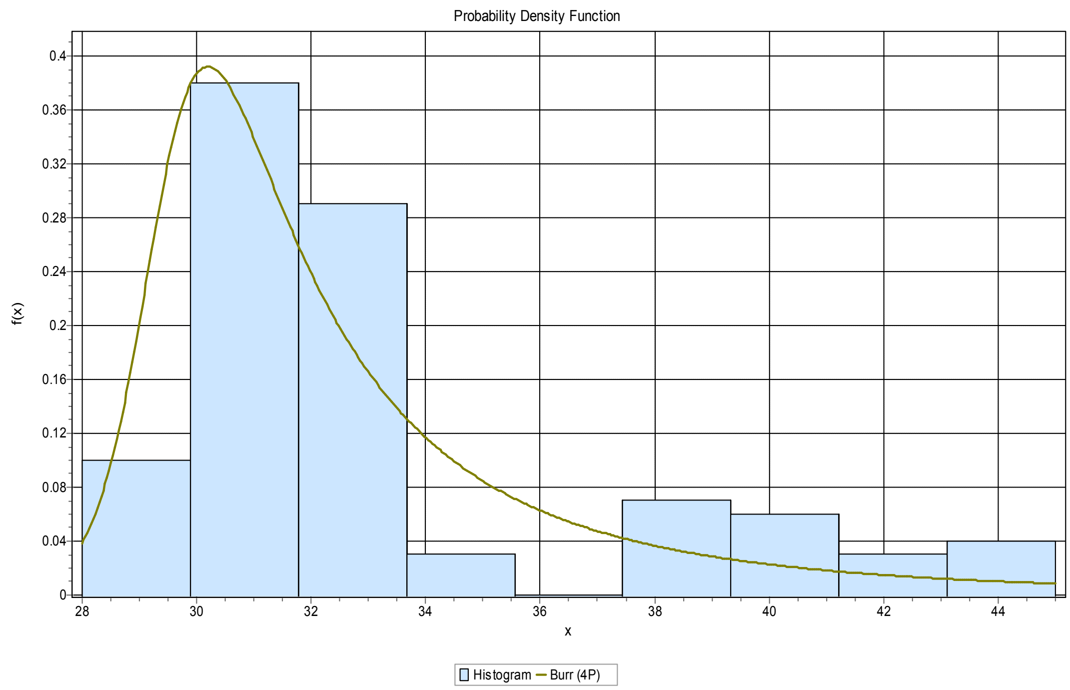

Fig. 3. Histogram of the time use for site survey

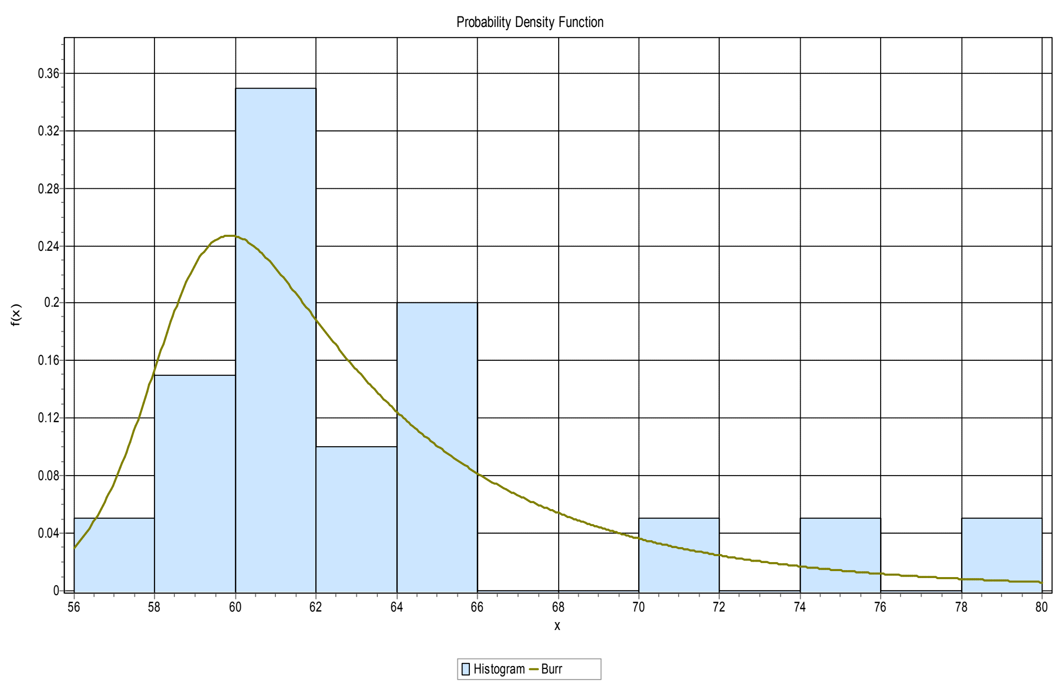

Fig. 4. Histogram for the time use for analysis of data

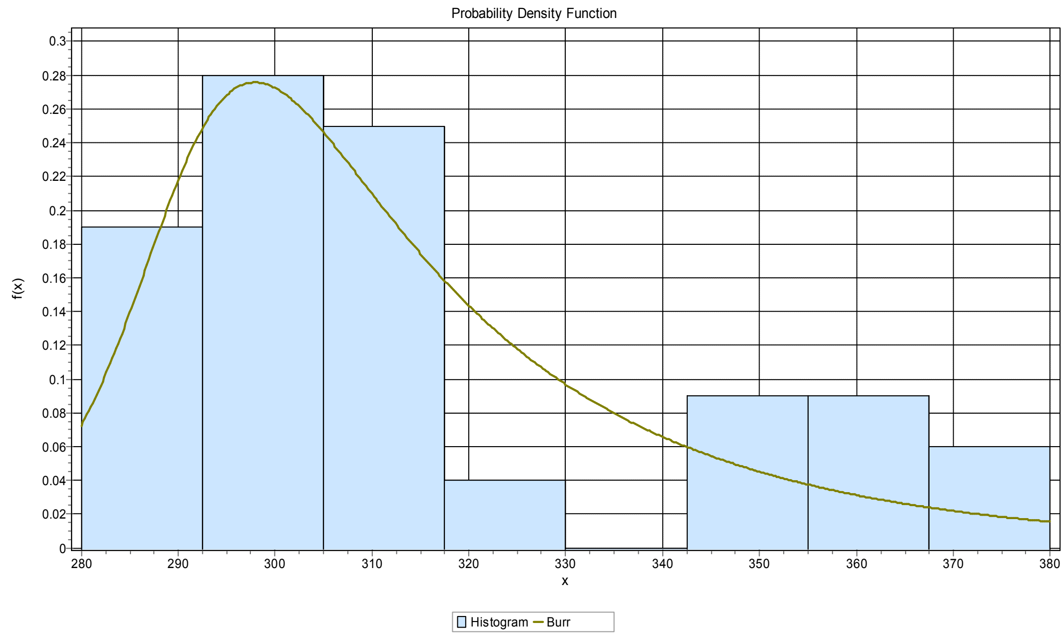

Fig. 5. Histogram of the time use for drilling

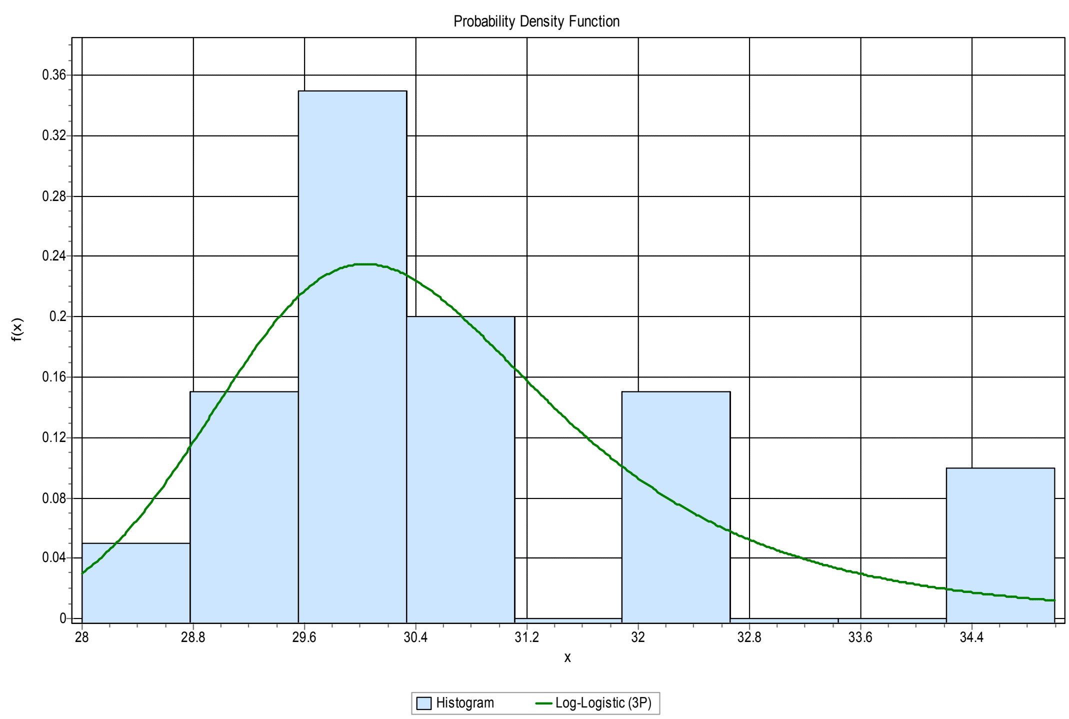

Fig. 6. Histogram of the time use for casing

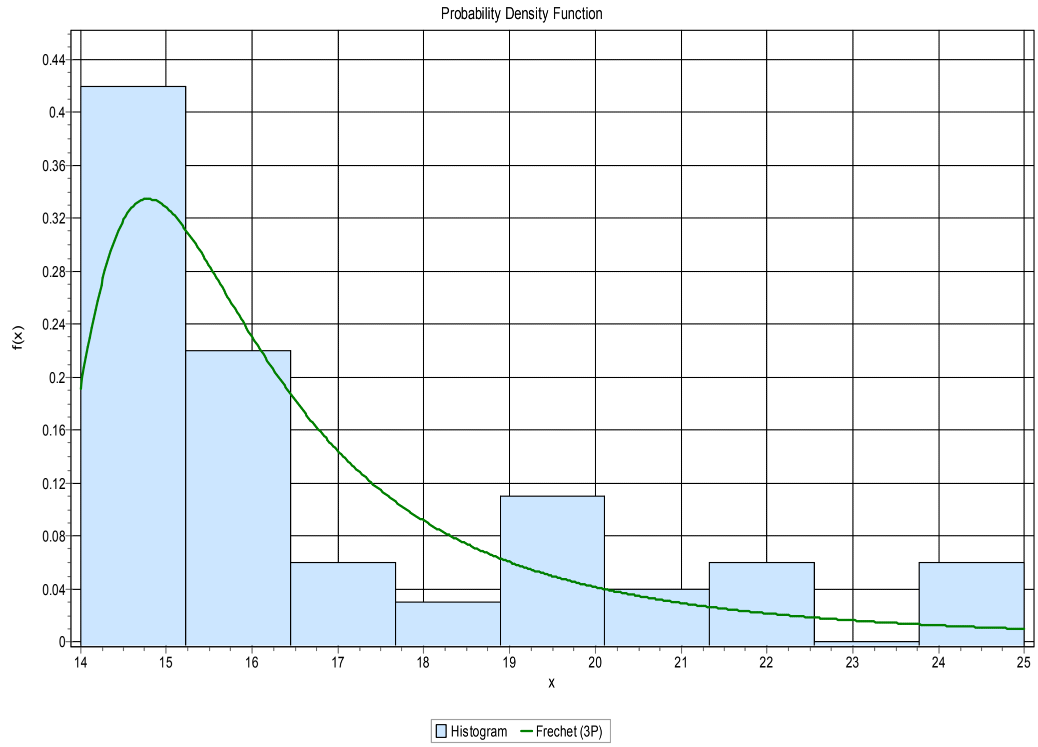

Fig. 7. Histogram of time use for gravel packing

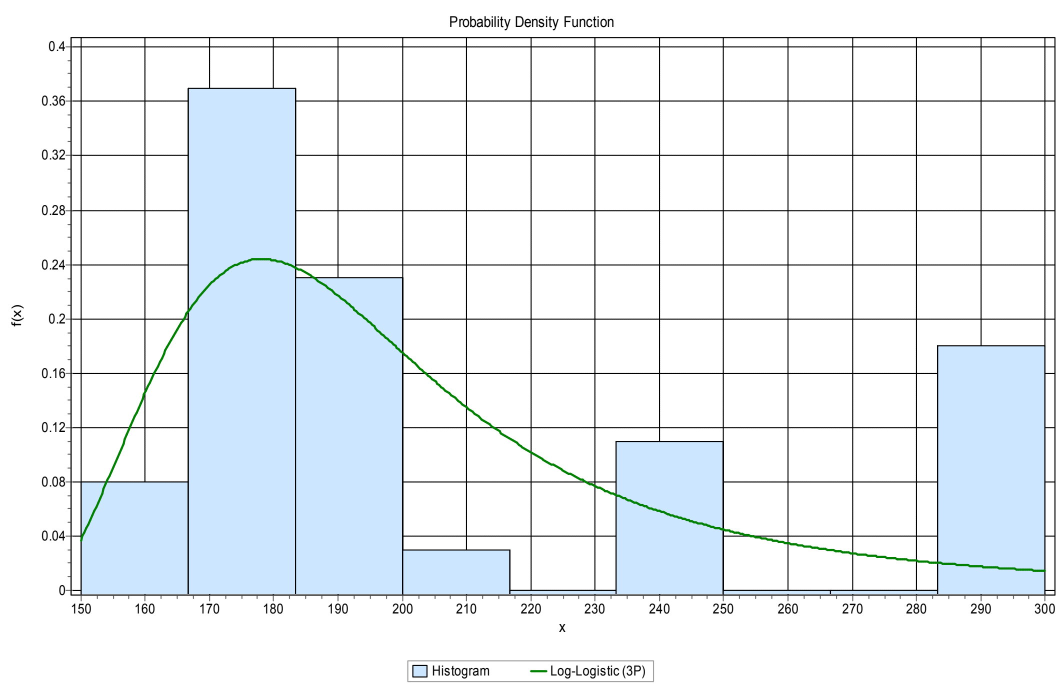

Fig. 8. Histogram of the time use for the connection of pipes to pump

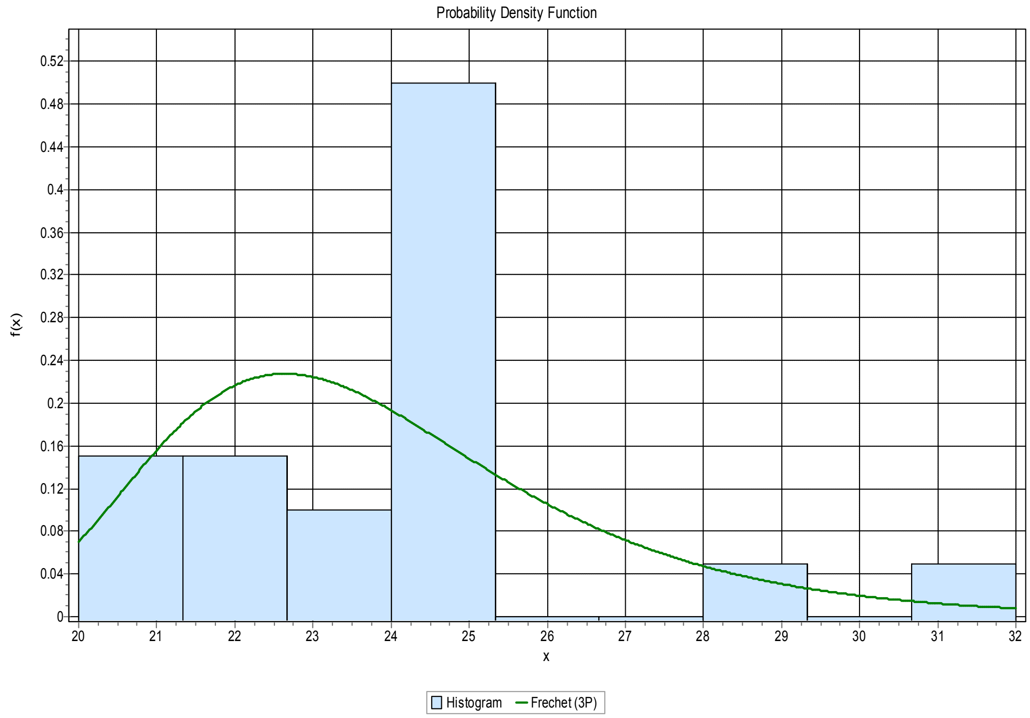

Fig. 9. Histogram of the time use for the connection of cables to pump

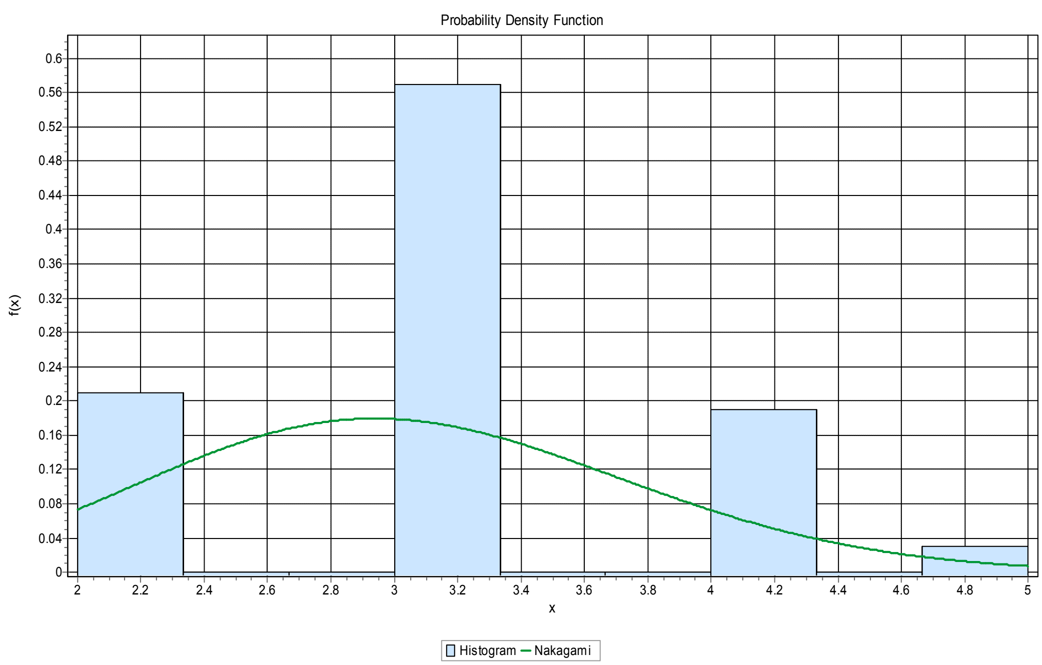

Fig. 10. Histogram for the time used for lowering of pump

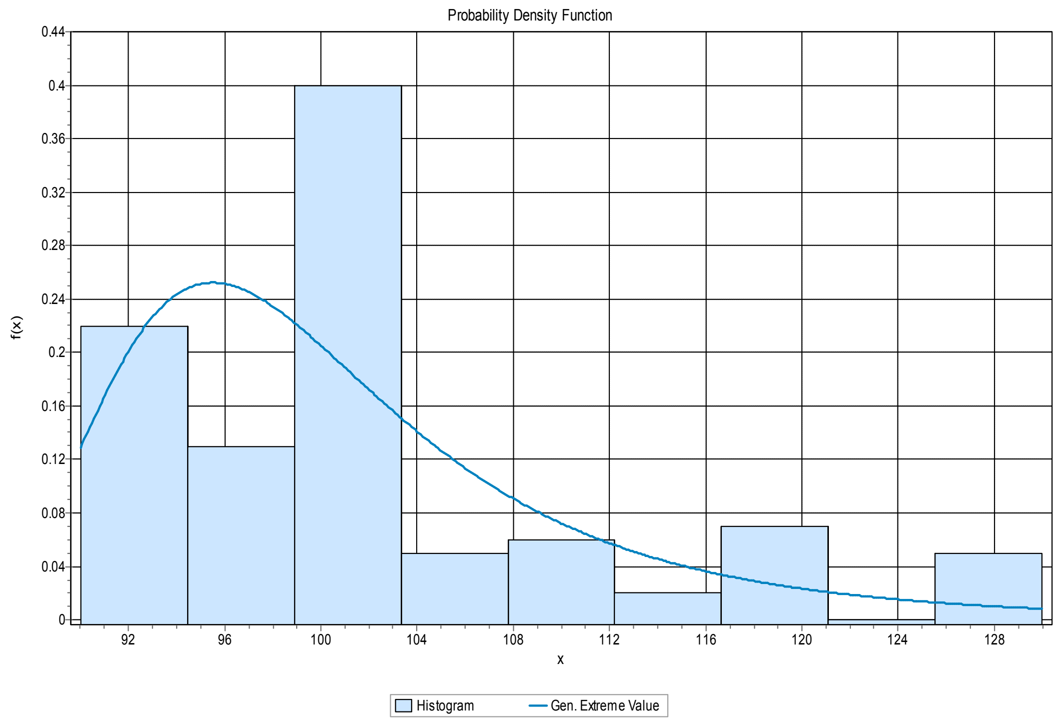

Fig. 11. Histogram of plumbing time

Out of the 9 activities Burr XII distribution made the best fit for three activities, namely, A-geographical survey, B-data analysis and C-punching. Log-logistic distribution; which is a special case of the Burr XII distribution made the best fit for activities D and F. Hence, Burr distribution can equally model activities D and F. Frechet distribution made the best fit for activities E and G. The Frechet distribution is an extreme value distribution. The general extreme value distribution, made the best fit for activity I. Recall, that Burr XII distribution region cover some sections of the extreme value distribution on the skewness-kurtosis plane. Nakagami distribution made the best fit for activity H. However, Kolmogorov Smirnov test ranked Dagum distribution 2nd and 3-parameter Burr XII distribution 3rd after Nakagami distribution. If a random variable X has Burr XII distribution, then $1/X$ has a Burr III distribution which is sometimes referred to as Dagum distribution [27]. Hence, the Dagum distribution belongs to the family of Burr distributions [26].

On the whole, the kolmogorov Smirnov test, Anderson Darling test, and Chi-square tests affirm that Burr XII fits the activity duration data for all 9 activities.

From the foregoing, we conclude that Burr XII distribution can be chosen as an input distribution for activity duration.

4 Simulation of Project Completion Time

The network that describes the water borehole project is presented in Fig. 12. By observation we notice that the project network has two paths, namely $1-2-3-4-5-6-7-8-9$ and $1-2-3-4-5-6-7-9$ that connects the start and the end node. Upon realization of the network, which simply means the application of the critical path algorithm to a long series of realizations (in our case 1000); each one obtained by assigning a sample value to every activity drawn from the Burr XII density. The distribution function of the project duration T is then obtained. This process defines the Monte Carlo simulation technique.

Let the completion time of all activities on j^{th} path be a random variable X_j defined as

$$X_j = \sum_{i \in P_j} T_i$$

where T_i represent durations of activities in the project. The critical path is given as the longest path in the network and its length is the completion time of the project.

Let T represent the the project completion time then

$$T = \text{Max}(X_j)\ ;\ \text{for all } j.$$

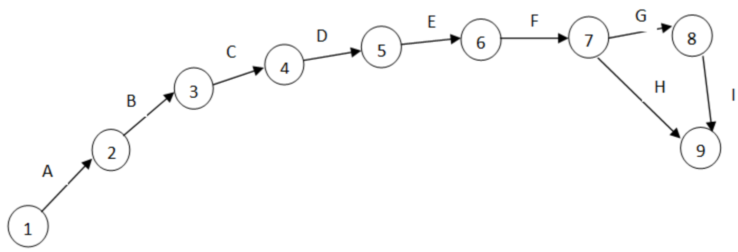

Fig. 12. Network Diagramme for bore hole drilling project

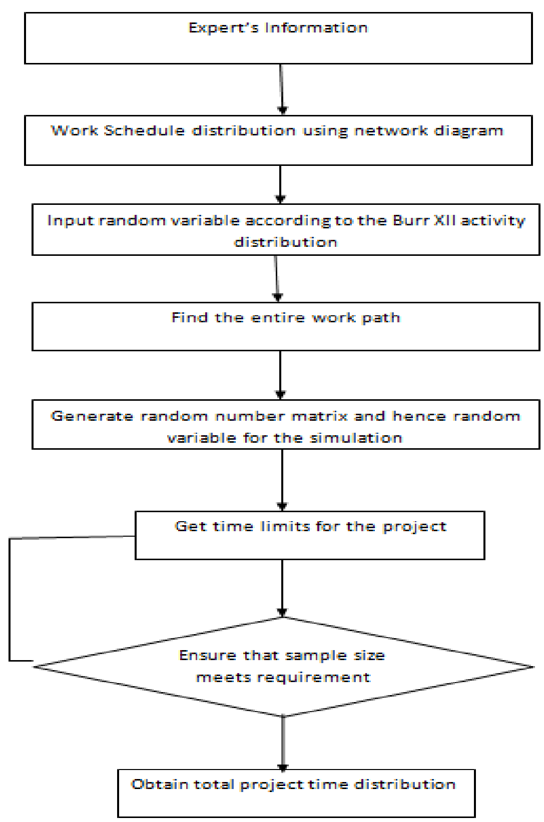

Fig. 13. Flow chart for Monte Carlo simulation of project network

Fig. 13 presents the flow chart for the Monte-Carlo Simulation. A programme written in MATLAB codes was used for the Monte Carlo Simulation. From the simulation results path 1 is critical with mean =796.2736 minutes and variance = 2751.9866. The distribution of project completion time is presented in Fig. 14. Meanwhile, simulation results (using RiskAMP^{TP}) with PERT-Beta distribution gives the estimate of project completion time as 775.6400 minutes with variance 979.7600 minutes. A comparison of these simulation results (using Burr XII distribution and the PERT-Beta distribution) give errors of approximately 3% and 64% for the mean and variance respectively. The large error in the variance estimate is certain because PERT-Beta distribution measures the dispersion of the distribution as a constant function of the range [24].

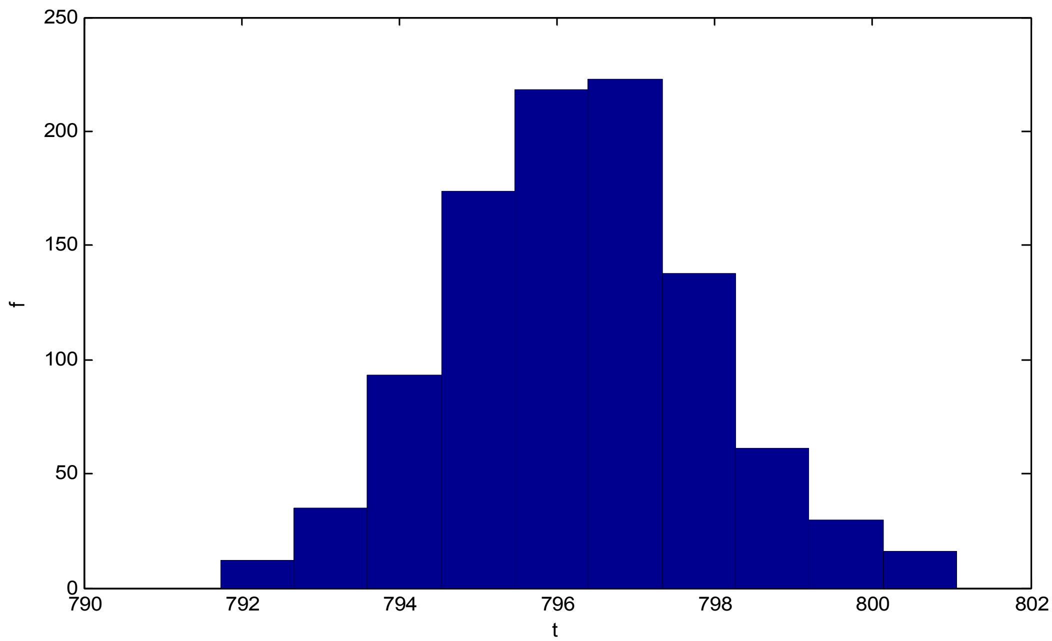

Fig. 14. Histogram showing the distribution of project completion time

5 Conclusion

A pilot case study of water borehole drilling project revealed that some distribution functions which are not 'popular' qualify as best-fit distributions for modelling activity durations in stochastic network analysis and many simulation software (example Devize, @Risk) do not include these set of distributions as default. In this paper, Burr XII distribution was used as an input activity duration distribution for Monte Carlo Simulation of PERT Networks. The choice of Burr XII distribution was based on its robustness, simplicity and its ability to model many real life scenarios. A comparison of the results obtained using Burr XII activity distribution with that of PERT-Beta reveals that the later model under estimate the project mean and variance. Our results is based on a simple project network with only 9 activities. Hence, we suggest that a more complex network with many activities be used to verify these results.

In Monte Carlo simulation, quantile functions are often used to generate pseudo-random numbers by the popular inverse transform method. However, if the distribution function of the underlying activity duration is not in closed form, the simulation process could be a tedious task. Interestingly, Burr XII distribution possesses this very important characteristic. Hence, given some information by experts, the parameters of the Burr XII distribution can easily be elicited using quantile estimation approach and then a Monte Carlo simulation performed.

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Competing Interests

Authors have declared that no competing interests exist.

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