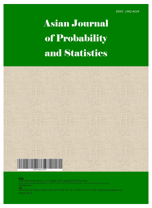

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Modeling Life Time Data by Generalized Weibull-generalized Exponential Distribution

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Authors' contributions

This work was carried out in collaboration between both authors. Author NIB designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors NIB and FO managed the analyses of the study. Author NIB managed the literature searches. Both authors read and approved the final manuscript.

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Abstract

This paper convolutes two generalized distributions from the family of generated T - X distribution. The new distribution generated from these distributions is called the Generalized Weibull-generalized Exponential Distribution. The properties of the proposed distribution are derived. Method of maximum likelihood estimation is used to estimate the parameters of the distribution and the information matrix is obtained. Thereafter, the distribution is applied to a real life dataset of failure for the air conditioning system and the obtained results are compared with other existing distributions to illustrate the capability and flexibility of the new distribution.

Keywords: Asymptotic behaviour; generating function; moments; T – X distribution; information matrix.

1 Introduction

In literature, researchers have developed several distributions which are flexible in nature and good for fitting specific real life datasets using different methods namely: Logit of beta link function, generator

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approach, exponential T-X. All of these methods were used to add one parameter or more into parent/baseline distribution. [1] introduced the beta link function to convolute two or more distributions to have a better distribution than the parent distribution. [2] developed beta Weibull distribution. Extended Beta-Pareto distribution was introduced by [3]. [4] worked on beta-exponential distribution. [5] worked on the beta weighted exponential distribution. The beta modified weighted Rayleigh distribution was also developed by [6] and so on.

Furthermore, generator approach, which was used to introduce beta Normal distribution, was initiated by [7]. [8] and [9] used this same approach in their studies according to the literature. The exponentiated T-X approach was pioneered by [10] to propose a new method for generating family of continuous distributions. [11] used the method in their work. Hence, [12] introduced the generalized Weibull-exponential distribution based on the method. In this work, we also use the exponentiated T-X approach to define a new distribution namely: the generalized Weibull-generalized exponential (GWGE) distribution.

The paper is arranged as follows: section two contains the GWGE distribution. In section three, we derive the following properties: distribution function, reliability function, hazard rate function, asymptotic behaviour, moments, generating function, skewness, kurtosis, estimation parameter and information matrix. Sections four and five contain the application of the distribution to a real data set and the conclusion.

2 Materials and Methods

2.1 The generalized Weibull-Generalized Exponential (GWGE) distribution

Let X be a continuous random variable which follows the generalized exponential distribution by [13] with density function given as:

$$f_{\{GE\}\{(x;\beta,\mu)\}} = \beta\theta\left(1 - e^{(-\theta(x))}\right)^{\beta-1} e^{(-\theta(x)} \tag{1}$$

and the corresponding distribution function is

$$F_{\{GE\}\{(x;\beta,\mu)\}} = \left(1 - e^{(-\theta(x))}\right)^{\beta} \tag{2}$$

where, $x > 0,\ \beta,\ \theta > 0.$

The exponentiated T-X distribution by [11] applied in [12] is given below as:

$$f(x) = \frac{c\,\alpha}{\gamma}\frac{f(x)F^{\{c-1\}}(x)}{1-F^{\{c\}}(x)}\frac{\left[\left\{-\ln\left(1-F^{\{c\}}(x)\right)\right\}\right]^{\{\alpha-1\}}}{\gamma}.\{\exp\}\left[-\left(\frac{\{-\ln\left(1-F^{\{c\}}(x)\right)\}}{\gamma}\right)\right]^{\{\alpha\}} \quad \alpha, c, \gamma \text{ and } x > 0 \tag{3}$$

We define the density function of the GWGE distribution by substituting equations (1) and (2) into (3) as a mixture/convolution of two existing distributions to obtain flexible, versatile and a good description of real life data. Here, the probability density function of the GWGE distribution is defined as:

$$f_{\mathrm{GWGED}}(x) = \frac{c\,\alpha}{\gamma}\,\frac{\beta\theta e^{-\theta(x)}\left(1-e^{-\theta(x)}\right)^{\beta(c+1)-1}}{1-(1-e^{-\theta(x)})^{\beta c}}\left[\frac{-\ln\{(1-((1-e^{-\theta(x)})^{\beta c}}{\gamma}\right]^{\alpha-1}.$$
$$\exp-\left[\left(\frac{-\ln\{(1-((1-e^{-\theta(x)})^{\beta c}}{\gamma}\right)\right]^{\alpha}, \beta, \theta > 0 \tag{4}$$

where α, β and c are shape parameters and θ, and γ are scale parameters respectively. Some special cases were emanated from the distribution when one or more parameter(s) equal to one. For instance; if:

(i) α= c = 1; the GWGE distribution yields generalized exponential (GE) distribution with parameters $\left(\frac{\gamma}{\theta}, \beta\right)$
(ii) $\alpha = \gamma = 1$; it reduces to exponentiated generalized exponential (EGE) distribution with parameters $\left(\frac{\gamma}{\theta}\right)$ [13,14].
(iii) c = 1; the GWGE distribution becomes generalized Weibull (GW) distribution with parameters $\left(\frac{\gamma}{\theta}, \alpha, \beta\right)$ [15].
(iv) $\beta = c = 1$; we have Weibull (W) distribution with parameters $\left(\frac{\gamma}{\theta}, \alpha\right)$
(v) $\alpha = \beta = \gamma = 1$; the GWGE distribution leads to exponentiated exponential (EE) distribution with parameters [14].
(vi) $\beta = 1$; GWGE distribution yields generalized Weibull-exponential (GWE) distribution with parameters $\left(\frac{\gamma}{\theta}, \alpha, c\right)$ [12].

By integrating (4) over the range of X, if X has the pdf in (4) and the associating distribution function is investigated as follows:

$$\mathrm{F}_{\{\mathrm{GWGED}\}}{}^{(\mathrm{x})} = \mathrm{P}(\mathrm{X} \le \mathrm{x}) = \int_0^{\mathrm{x}} \mathrm{f}_{\{\mathrm{GWGED}\}}{}^{(\mathrm{x})} dx \tag{5}$$

$$= \int_0^x \frac{\mathrm{c}\,\alpha}{\gamma} \frac{\beta\theta \mathrm{e}^{-\theta(\mathrm{x})}\left(1-\mathrm{e}^{-\theta(\mathrm{x})}\right)^{\beta(\mathrm{c}+1)-1}}{1-\left(1-\mathrm{e}^{-\theta(\mathrm{x})}\right)^{\beta\mathrm{c}}} \left[\frac{-\ln\{(1-((1-\mathrm{e}^{-\theta(\mathrm{x})})^{\beta\mathrm{c}}}{\gamma}\right]^{\alpha-1} \exp - \left[\left(\frac{-\ln\{(1-((1-\mathrm{e}^{-\theta(\mathrm{x})})^{\beta\mathrm{c}}}{\gamma}\right)\right]^{\alpha} \tag{6}$$

Since the distribution function of exponential distribution is $1 - \mathrm{e}^{-\theta(\mathrm{x})}$ therefore the cumulative density function (cdf) of the GWGE distribution is given as

$$F_{\{GWGED\}}(x) = 1 - \{exp\}\left[-\left(\frac{-\ln\left(1-\left\{\left(1-e^{-\theta\{x\}}\right)^{\{\beta\{c\}\}}\right\}\right)}{\gamma}\right)\right]^{\alpha} \tag{7}$$

The Reliability function of GWGE distribution is

$$R_{\{GWGED\}}(x) = 1 - F_{\{GWGED\}\{(x)\}} = 1 - \int_0^x f_{\{GWGED\}}(x)dx \tag{8}$$

$$= 1 - \left[1 - \{exp\}\left[-\left(\frac{-\ln\left(1-\left(1-e^{-\theta\{x\}}\right)^{\beta\{c\}}\right)}{\gamma}\right)\right]^{\alpha}\right] \tag{9}$$

$$R_{\{GWGED\}}(x) = \{exp\}\left[-\left(\frac{-\ln\left(1-\left(1-e^{\{-\theta\{x\}\}}\right)^{\beta\{c\}}\right)}{\gamma}\right)\right]^{\alpha}, x > 0 \tag{10}$$

While, the hazard rate function is given as:

$$\mathrm{HD}_{\{\mathrm{GWGED}\}}(\mathrm{x}) = \frac{\mathrm{f}_{\{\mathrm{GWGED}\}}(\mathrm{x})}{1-\mathrm{F}_{\{\mathrm{GWGED}\}}(\mathrm{x})} = \frac{\mathrm{f}_{\{\mathrm{GWGED}\}}(\mathrm{x})}{\mathrm{R}_{\{\mathrm{GWGED}\}}(\mathrm{x})} \tag{11}$$

by dividing the pdf and cdf or reliability function as we have it in (11) above, part of the equation will reduce the equation and the hazard function of GWGE distribution is given in the expression (12) below

$$HD_{\{GWGED\}}(x) = \frac{c\,\alpha}{\gamma}\frac{\beta\theta e^{-\theta(x)}\left(1-e^{-\theta(x)}\right)^{\beta(c+1)-1}}{1-\left(1-e^{-\theta(x)}\right)^{\beta c}}\left[\frac{-\ln\{(1-((1-e^{-\theta(x)})^{\beta c}}{\gamma}\right]^{\alpha-1} \quad (12)$$

The plots of the density function and hazard function are depicting in figure 1 below for different values of the parameters

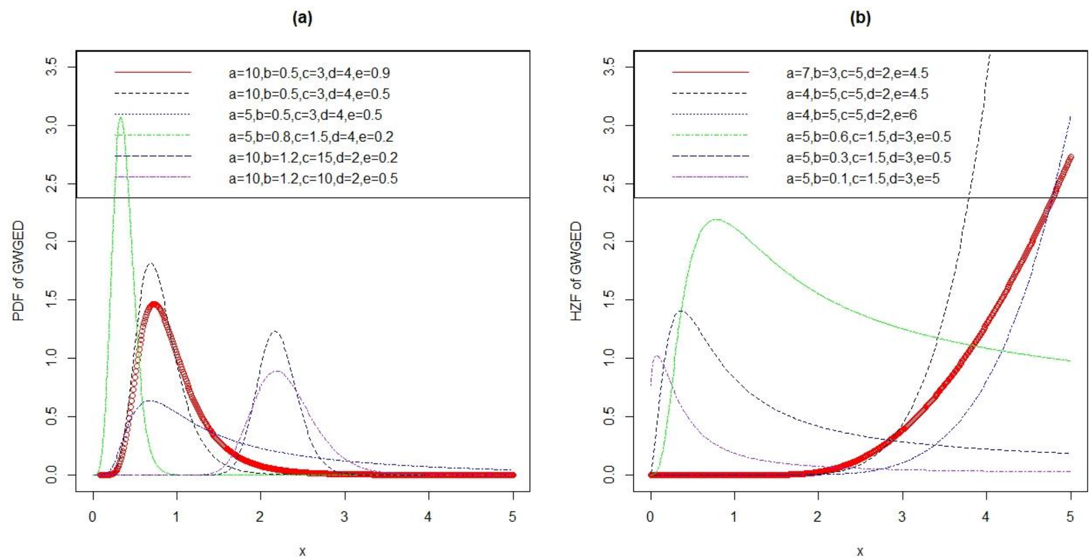

Fig. 1. Plots of the GWGE distribution densities in (a) and hazard rate function in (b) . Where a=α (shape 1 parameter), b=β (shape 2 parameter), c=c (shape 3 parameter), d=θ (scale 1 parameter) and e=γ (scale 2 parameter)

3 Properties of the GWGED

3.1 The asymptotic properties of the PDF and hazard function of GWGED

Here, we examine the asymptotic properties of the GWGE distribution by study its behaviour as x tends to infinity and as x tends to zero by investigating the limiting behaviour of the distribution.

$$\lim_{x\to\infty} f_{GWGED}(x) = lim_{x\to\infty}\frac{c\,\alpha}{\gamma}\frac{\beta\theta e^{-\theta(x)}\left(1-e^{-\theta(x)}\right)^{\beta(c+1)-1}}{1-(1-e^{-\theta(x)})^{\beta c}}\left[\frac{-\ln\{(1-((1-e^{-\theta(x)})^{\beta c}}{\gamma}\right]^{\alpha-1}.$$
$$\exp-\left[\left(\frac{-\ln\{(1-((1-e^{-\theta(x)})^{\beta c}}{\gamma}\right)\right]^{\alpha} \quad (13)$$

Meanwhile, by substituting infinity ∞ into x, the expression is $lim_{x\to\infty}\frac{c\,\alpha}{\gamma}\frac{\beta\theta e^{-\theta(x)}\left(1-e^{-\theta(x)}\right)^{\beta(c+1)-1}}{1-\left(1-e^{-\theta(x)}\right)^{\beta c}} = 0$, and with $x \to 0$; the limit is $\lim_{x\to 0}\frac{c\,\alpha}{\gamma}\frac{\beta\theta e^{-\theta(x)}\left(1-e^{-\theta(x)}\right)^{\beta(c+1)-1}}{1-\left(1-e^{-\theta(x)}\right)^{\beta c}} = 0$. Also, for hazard function $\lim_{x\to\infty} HD_{\{GWGED\}}(x) = \lim_{x\to 0} HD_{\{GWGED\}}(x) = 0$. This indicates that GWGE distribution has at least one mode. It implies that the density function of the GWGE distribution is unimodal that is any peak is a mode.

3.2 Moments

We derive the s-th moments of the GWGED via the moment generating function (mgf) as:

$$M_{\{x\}}(t) = E\left(e^{(tx)}\right) = \int_0^\infty e^{(tx)} f_{\{GWGED\}}(x)dx \tag{14}$$

$$= \int e^{\{tx\}} \frac{c\,\alpha}{\gamma} \frac{\beta\theta e^{-\theta(x)}\left(1-e^{-\theta(x)}\right)^{\beta(c+1)-1}}{1-(1-e^{-\theta(x)})^{\beta c}} \left[\frac{-\ln\{(1-((1-e^{-\theta(x)})^{\beta c}}{\gamma}\right]^{\alpha-1}.$$

$$\exp -\left[\left(\frac{-\ln\{(1-((1-e^{-\theta(x)})^{\beta c}}{\gamma}\right)\right]^{\alpha} dx$$

set $K = \left[-\left(\frac{-\ln\{(1-((1-e^{-\theta(x)})^{\beta c}}{\gamma}\right)\right]^{\alpha}$

Then,

$$= \int_0^\infty e^{\{tx\}} \frac{c\,\alpha}{\gamma} \frac{\beta\theta e^{-\theta(x)}\left(1-e^{-\theta(x)}\right)^{\beta(c+1)-1}}{1-(1-e^{-\theta(x)})^{\beta c}} K^{-1}.e^{k}\,dx$$

$$= \frac{c\,\alpha}{\gamma} \frac{\beta\theta e^{-\theta(x)}\left(1-e^{-\theta(x)}\right)^{\beta(c+1)-1}}{1-(1-e^{-\theta(x)})^{\beta c}} K^{-1} = \int_0^\infty e^{\{tx\}}.e^{k}\,dx$$

where, $\int_0^\infty e^{\{tx\}}.e^{k}\,dx = 1$. Therefore, above expression yields:

$$dx = \frac{\gamma\left(1-\left(1-e^{-\theta(x)}\right)^{\beta c}\right)}{c\,\alpha\beta\theta e^{-\theta(x)}(1-e^{-\theta(x)})^{\beta\{(c+1)-1}[-\ln(1-((1-e^{-\theta(x)})^{\beta c}]}dk$$

Furthermore, expression (14) can be written as:

$$M_{\{x\}}(t) = \int_0^\infty e^{k} \frac{t}{(1-(1-e^{-\theta(x)})^{(\beta c)-1})} dk$$

$$M_x(t) = 1 + \sum_{j=0}^{\infty} \frac{(t)_j}{j!}\left(\sum_{n=0}^{\infty} \frac{(i)^j}{n!}\left(\frac{j}{c}\right)_n \sum_{m=0}^{\infty} \frac{(i)^m n^m}{n!}\left(\frac{\gamma}{\theta}\right)^m \Gamma\left(1+\frac{m}{\alpha\beta\theta}\right)\right) \tag{15}$$

where $(t)_{(j)} = t(t+1)\dots(t+j-1)$, as stated in [12]. Hence, taking the s-th derivative of (15) and t = 0 the sth moments is gives as:

$$E(X^s) = \sum_{j=0}^{\infty} \frac{d^s}{dt^s}\frac{(t)_j}{j!}\left(\sum_{n=0}^{\infty} \frac{(i)^j}{n!}\left(\frac{j}{c}\right)_n \sum_{m=0}^{\infty} \frac{(i)^m n^m}{n!}\left(\frac{\gamma}{\theta}\right)^m \Gamma\left(1+\frac{m}{\alpha\beta\theta}\right)\right) \tag{16}$$

The expected value of GWGED is:

$$E(X) = \sum_{j=0}^{\infty} \frac{1}{j}\left(\sum_{n=0}^{\infty} \frac{(i)^j}{n!}\left(\frac{j}{c}\right)_n \sum_{m=0}^{\infty} \frac{(i)^m n^m}{n!}\left(\frac{\gamma}{\theta}\right)^m \Gamma\left(1+\frac{m}{\alpha\beta\theta}\right)\right) \tag{17}$$

Also, the associating variance of the GWGED is:

$$Var(X) = \sum_{j=0}^{\infty} \frac{2(-\omega(1)+\omega(j))}{j}\left(\sum_{n=0}^{\infty} \frac{(i)^j}{n!}\left(\frac{j}{c}\right)_n \sum_{m=0}^{\infty} \frac{(i)^m n^m}{n!}\left(\frac{\gamma}{\theta}\right)^m \Gamma\left(1+\frac{m}{\alpha\beta\theta}\right)\right) \tag{18}$$

While the quantile function, skewness and kurtosis of the GWGED are given respectively as follows:

$$Q(\lambda) = -\ln(1 - \left[1 - e^{-\left(\frac{-\ln(1-\theta\lambda)}{\gamma}\right)^{\alpha-1}}\right]^{(\beta c)^{-1}}$$

Following the quantile in [10,11,12] by setting $\lambda = \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\right)$. The skewness and kurtosis of the GWGED are:

$$\mathrm{SK}_{\{\mathrm{GWGED}\}}(\mathrm{x}) = \frac{Q(0.75) - 2Q(0.5) + Q(0.25)}{Q(0.75) - Q(0.25)} \tag{19}$$

and

$$\mathrm{KUR}_{\{\mathrm{GWGED}\}}(\mathrm{x}) = \frac{Q(0.875) - Q(0.625) + Q(0.375) - Q(0.125)}{Q(0.75) - Q(0.25)} \tag{20}$$

3.3 Parameter estimation and information matrix

We consider the five-parameter GWGE distribution and re-written $\lambda = \gamma^{-1}$ for easy mathematics: $\lambda c\alpha\beta\theta e^{-\theta x}\left(1 - e^{-\theta x}\right)^{\beta-1}\left(1 - e^{-\theta x}\right)^{-1} exp\left[-\left\{-\lambda ln\left(1 - \left(1 - e^{-\theta x}\right)^{\beta c}\right)\right\}\right]^{\alpha}$.

Invariably, we then denote the MLEs of $\alpha, \beta, \theta, c, \lambda$ as $\hat{\alpha}, \hat{\beta}, \hat{\theta}, \hat{c}, \hat{\lambda}$ respectively. Let $X_{(1)}, \dots, X_{(n)}$ be a random sample of size n from GWGED $\alpha, \beta, \theta, c, \lambda$ therefore the log-likelihood function $L(\alpha, \beta, \theta, c, \lambda)$ is

$$L(\alpha, \beta, \theta, c, \lambda) = nlog(\alpha) + nlog(\beta) + nlog(\theta) + nlog(c) + nlog(\lambda) - \sum_{i=1}^{n} \theta(x_i)$$
$$+(\beta - 1)\sum_{i=1}^{n} \log\left(1 - e^{-\theta x}\right) - \sum_{i=1}^{n} \log\left(1 - e^{-\theta x}\right) + (\alpha - 1)\lambda ln \sum_{i=1}^{n} \log(1 - \left(1 - e^{-\theta x}\right)^{\beta c})$$
$$-\alpha\lambda ln \sum_{i=1}^{n} \log(1 - \left(1 - e^{-\theta x}\right)^{\beta c}) \tag{21}$$

Taking first partial derivative in (21) with respect to $\alpha, \beta, \theta, c, \lambda$ and equating to zero, we obtain the normal equations as follows:

$$\frac{\partial L}{\partial \alpha} = \frac{n}{\alpha} + \lambda ln \sum_{i=1}^{n} \log(1 - \left(1 - e^{-\theta x}\right)^{\beta c}) - \lambda ln \sum_{i=1}^{n} \log\left(1 - \left(1 - e^{-\theta x}\right)^{\beta c}\right) = 0 \tag{22}$$

$$\frac{\partial L}{\partial \beta} = \frac{n}{\beta} + \sum_{i=1}^{n} \log\left(1 - e^{-\theta x}\right) + (\alpha - 1)c\lambda ln \sum_{i=1}^{n} \log(1 - \left(1 - e^{-\theta x}\right)^{\beta c})$$
$$+\alpha c\lambda ln \sum_{i=1}^{n} \log\left(1 - \left(1 - e^{-\theta x}\right)^{\beta c}\right) = 0 \tag{23}$$

$$\frac{\partial L}{\partial \theta} = \frac{n}{\theta} nc + n\beta + (\beta - 1)\sum_{i=1}^{n} \frac{1 - e^{-\theta x}}{\left(1 - e^{-\theta x}\right)} - (\alpha - 1)\alpha\beta c\lambda ln \sum_{i=1}^{n} \frac{1 - \left(1 - e^{-\theta x}\right)^{\beta c}}{\left(1 - \left(1 - e^{-\theta x}\right)^{\beta c}\right)} = 0 \tag{24}$$

$$\frac{\partial L}{\partial c} = \frac{n}{c} + (\alpha - 1)\beta\lambda ln \sum_{i=1}^{n} 1 - \left(1 - e^{-\theta x}\right)^{\beta c} - \alpha\beta\lambda ln \sum_{i=1}^{n} 1 - \left(1 - e^{-\theta x}\right)^{\beta c} = 0 \tag{25}$$

$$\frac{\partial L}{\partial \lambda} = \frac{n}{\lambda} + (\alpha - 1)\sum_{i=1}^{n} 1 - \left(1 - e^{-\theta x}\right)^{\beta c} - \alpha \sum_{i=1}^{n} 1 - \left(1 - e^{-\theta x}\right)^{\beta c} = 0 \tag{26}$$

We equally obtained the second derivatives of $L(\alpha,\beta,\theta,c,\lambda)$ in appendix below.

The Newton-Rapson algorithm can also be used to solve the non-linear equations simultaneously as we had considered the second derivative of $L(\alpha,\beta,\theta,c,\lambda)$. The variance-covariance (information matrix) $l^{-1}$is given as:

$$L(\alpha,\beta,\theta,c,\lambda)=E\begin{bmatrix}\frac{\partial^2 L}{\partial\alpha^2} & \frac{\partial^2 L}{\partial\alpha\partial\beta} & \frac{\partial^2 L}{\partial\alpha\partial c} & \frac{\partial^2 L}{\partial\alpha\partial\theta} & \frac{\partial^2 L}{\partial\alpha\partial\lambda}\\ \frac{\partial^2 L}{\partial\beta\partial\alpha} & \frac{\partial^2 L}{\partial\beta^2} & \frac{\partial^2 L}{\partial\beta\partial c} & \frac{\partial^2 L}{\partial\beta\partial\theta} & \frac{\partial^2 L}{\partial\beta\partial\lambda}\\ \frac{\partial^2 L}{\partial c\partial\alpha} & \frac{\partial^2 L}{\partial c\partial\beta} & \frac{\partial^2 L}{\partial c^2} & \frac{\partial^2 L}{\partial c\partial\theta} & \frac{\partial^2 L}{\partial c\partial\lambda}\\ \frac{\partial^2 L}{\partial\theta\partial\alpha} & \frac{\partial^2 L}{\partial\theta\partial\beta} & \frac{\partial^2 L}{\partial\theta\partial c} & \frac{\partial^2 L}{\partial\theta^2} & \frac{\partial^2 L}{\partial\theta\partial\lambda}\\ \frac{\partial^2 L}{\partial\lambda\partial\alpha} & \frac{\partial^2 L}{\partial\lambda\partial\beta} & \frac{\partial^2 L}{\partial\lambda\partial c} & \frac{\partial^2 L}{\partial\lambda\partial\theta} & \frac{\partial^2 L}{\partial\lambda^2}\end{bmatrix} \tag{27}$$

4 Application to Real Data

The data set consists of 118 numbers of successive failures for the air conditioning system of each member in a fleet of 13 Boeing 720 jet air planes as recorded in [16] and can be found in [17]. The data is used only for illustrative purposes and numerical analysis is done using R software.

Tables 1 and 2 below contain the descriptive statistics and the maximum likelihood estimates (MLEs) with associating standard errors in parentheses of the distribution parameters.

Table 1. Descriptive Statistics for failures of air conditioning system

Min	Q1	Median	Mean	Q3	Max	Skewness	Kurtosis
1.00	20.75	54.00	92.07	118.00	603.00	2.139207	8.23109

Table 2. MLEs of the distribution parameters, the associating SEs (given in parentheses) and the statistic-2LogL, AIC, BIC and CAIC

Distribution	Estimate					Statistic			
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{c}$	$\hat{\theta}$	$\hat{\lambda}$	$-2LogL$	AIC	BIC	$CAIC$
GWGED	**0.596 (3.823)**	**2.597 (0.262)**	**1.597 (1.900)**	**0.081 (0.000)**	**0.597 (1.881)**	**103103**	**206210.0**	**206228.9**	**206229.9**
GWED	0.556 (4.723)		1.556 (4.713)	0.045 (0.000)	3.056 (0.003)	231374	462751.7	462766.7	462767.7
EGED		1.621 (2.595)	1.622 (2.596)	0.028 (0.000)		182146	364295.2	364306.5	364307.5
EED			1.594 (0.132)	0.084 (0.000)		442200	884401.2	884408.7	884409.7
GWD	0.546 (0.092)	2.546 25.921		0.152 (0.000)	1.546 (0.093)	711896	1423795	1423810	1423811
GED				0.100 (0.000)	2.500 (1.112)	352106	704214.6	704222.1	704223.1
WD	0.596 (3.105)			0.094 (0.000)	2.596 (3.093)	542250	1084504	1084515	1084526
ED		3.631 (0.066)		0.185 (0.003)	2.631 (0.003)	200327	4006535	4006546	4006547

The values appeared in Table 1 are the descriptive statistics of the data set we used. Where min (1.00) is the minimum number in the observations in the data set, Q1(20.75) is the first quartile, median (54.00) is the middle observation, Q3 (118.00) the third quartile, max (603.00) the maximum observation, skewness (2.139207) and Kurtosis (8.23109) the peak of the observations which is above normal peak.

The values in the Table 2 above reflect the estimation of parameters and model selection criteria: AIC, BIC and CAIC which of course the values of the GWGED are smaller compared with those values of the other distributions. Therefore, the GWGED seems to be a very good representative distribution to these data. Meanwhile, plots of the estimated PDF and CDF of the GWGED and other distributions fitted to these data and are given in Fig. 2. Also, GWGED display superiority to the other distributions in terms of fitting.

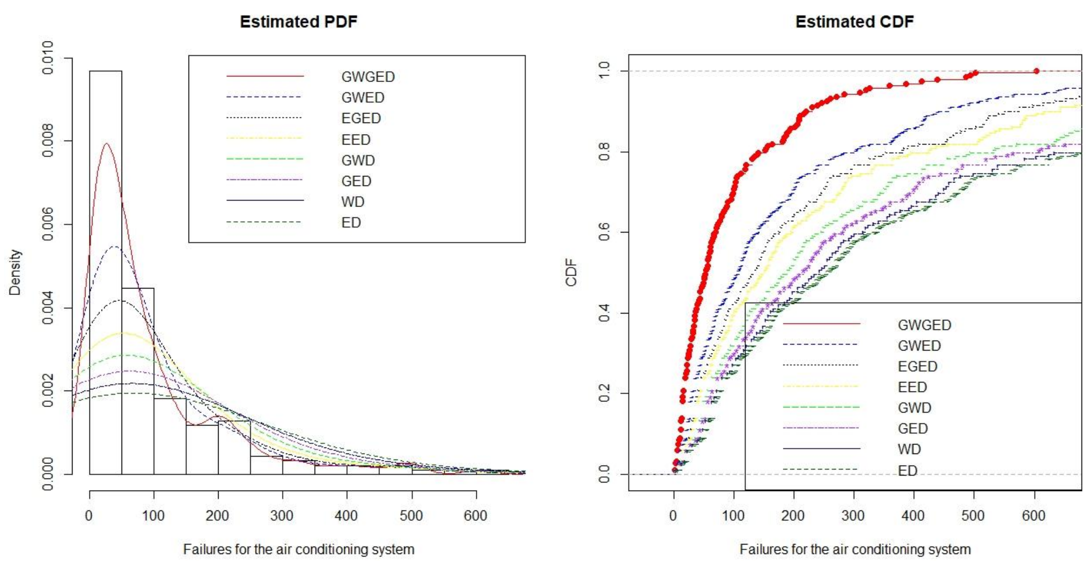

Fig. 2. Plots of the estimated PDF and CDF of all Models considered in the study

5 Conclusion

We are able to introduce a new distribution generated from exponentiated T-X approach called generalized Weibull-generalized exponential distribution by inserting weibull distribution into the exponentiated T-X by [11] and used in [12]. In the research work, some of the properties of the new distribution were presented. Hence, some known and unknown special cases such as: generalized Weibull-exponential distribution by [12], generalized exponential distribution by [13] and [14], generalized Weibull distribution [15] and exponentiated exponential distribution [14] and Weibull and exponential distribution were emanated from the new distribution. In the study, we also obtain the following: distribution function, reliability function, hazard rate function, asymptotic behaviour, moments, generating function, skewness and kurtosis. The distribution parameters were estimated using maximum likelihood and information matrix is investigated. An application to a real data set of 118 failures air conditioning system of each member in a fleet of 13 Boeing 720 jet air planes were used and the GWGE distribution proved superior to the fits of its major sub-distributions. Without doubt, the proposed GWGE distribution provides better fits to the data set than its sub-cases.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Jones MC. Families of distributions arising from distributions of order statistics test. 2004;13:1-43.

[2] Famoye F, Lee C, Olumolade O. The beta Weibull distribution. Journal of Statistical Theory and Applications. 2015;4:121-136.

[3] Akinsete A, Famoye F, Lee C. The Beta Pareto distribution. Statistics. 2008;42(6):547-563.

[4] Nadarajah S, Kotz S. The beta exponential distribution. Reliability Engineering and System Safety. 2006;91:689–697.

[5] Badmus NI, Bamiduro TA, Rauf-Animasaun MA, Akingbade AA. The beta weighted exponential distribution: Theory and applications. International Journal on Mathematical Analysis and Optimization: Theory and Application. 2015;55-66.

[6] Badmus NI, Alakija TO, Olanrewaju GO. The beta-modified weighted Rayleigh distribution: Application to virulent tubercle disease. International Journal on Mathematical Analysis and Optimization: Theory and Application. 2017;154–67.

[7] Eugene NC, Lee ET, Famoye F. Beta-normal distribution and its applications. Communications in Statistics Theory and Methods. 2002;31:497-512.

[8] Dey AK, Kundu D. Discriminating between the log-normal and log-logistic distributions. Communications in Statistics - Theory and Methods. 2010;39:280-292.

[9] Ashkar F, Mahdi S. Fitting log-logistic distribution by generalized moments. Journal of Hydrology. 2006;328:694-703.

[10] Alzaatreh A, Carl Lee, Felix F. A new method for generating families of continuous distributions. METRON3. 2013;71:63–79.

[11] Alzaghal A, Felix F, Carl L. Exponentiated T-X family of distributions with some applications. International Journal of Statistics and Probability. 2013;2(3):31–49.

[12] Hamdy MS, Mahmoud AS. The generalized Weibull-exponential distribution: Properties and applications. International Journal of Statistics and Applications. 2014;4(2):102–112.

[13] Gupta RD, Kundu D. Generalized exponential distribution. Australia. N. Z. Journal of Statistics. 1999;41:173–188.

[14] Gupta RD, Kundu D. Exponentiated exponential family: An alternative to gamma and Weibull distributions. Biometrical Journal. 2001;43:117-130.

[15] Mudholkar S, Kollia D. Generalized Weibull family: A structural analysis. Communications in Statistics - Theory and Methods. 1994;23(4):1149-1171.

[16] Proschan F. Theoretical explanation of observed decreasing failure rate. Technometrics. 1963;30(1):50-64.
DOI: https://doi.org/10.1080/00401706.1963.10490105

[17] Kumar D, Kumar M. A new generalization of the extended exponential distribution with an application. Annals of Data Science. 2018;1-22.
DOI: https://doi.org/10.1007/s40745-018-0180-0

Appendix

These are the second derivatives of $L(\alpha,\beta,\theta,c,\lambda)$

$$\frac{\partial^2 L}{\partial\alpha^2}=\frac{n}{\alpha^2}$$

$$\frac{\partial^2 L}{\partial\alpha\partial\beta}=\sum_{i=1}^{n}\frac{\left(e^{-\theta x}\right)^{c}\left(1-e^{-\theta x}\right)^{\beta c}}{1-(1-e^{-\theta x})^{\beta c}}-\sum_{i=1}^{n}\frac{\left(e^{-\theta x}\right)^{c}\left(1-e^{-\theta x}\right)^{\beta c}}{1-(1-e^{-\theta x})^{\beta c}}$$

$$\frac{\partial^2 L}{\partial\alpha\partial\theta}=\lambda ln\sum_{i=1}^{n}\frac{\left(1-e^{-\theta x}\right)^{\beta c}}{1-(1-e^{-\theta x})^{\beta c}}-\lambda ln\sum_{i=1}^{n}\frac{\left(1-e^{-\theta x}\right)^{\beta c}}{1-(1-e^{-\theta x})^{\beta c}}$$

$$\frac{\partial^2 L}{\partial\alpha\partial c}=\sum_{i=1}^{n}\frac{\left(c\left(e^{-\theta x}\right)^{\beta}-1\right)\left(1-e^{-\theta x}\right)^{\beta c}}{1-(1-e^{-\theta x})^{\beta c}}-\sum_{i=1}^{n}\frac{\left(c\left(e^{-\theta x}\right)^{\beta}-1\right)\left(1-e^{-\theta x}\right)^{\beta c}}{1-(1-e^{-\theta x})^{\beta c}}$$

$$\frac{\partial^2 L}{\partial\alpha\partial\lambda}=\sum_{i=1}^{n}\frac{\left(1-e^{-\theta x}\right)^{\beta c}}{1-(1-e^{-\theta x})^{\beta c}}-\sum_{i=1}^{n}\frac{\left(1-e^{-\theta x}\right)^{\beta c}}{1-(1-e^{-\theta x})^{\beta c}}$$

$$\frac{\partial^2 L}{\partial\beta^2}=\frac{n}{\beta^2}+(\alpha-1)c\lambda ln\sum_{i=1}^{n}\frac{\left(e^{-\theta x}\right)^{2}c\left(\left(1-e^{-\theta x}\right)^{\beta c}\right)}{\left(1-(1-e^{-\theta x})^{\beta^2 c}\right)}-\alpha c\lambda ln\sum_{i=1}^{n}\frac{\left(e^{-\theta x}\right)^{2}c\left(\left(1-e^{-\theta x}\right)^{\beta c}\right)}{\left(1-(1-e^{-\theta x})^{\beta^2 c}\right)}$$

$$\frac{\partial^2 L}{\partial\beta\partial\theta}=n+\sum_{i=1}^{n}\frac{\left(e^{-\theta x}\right)}{\left(1-e^{-\theta x}\right)}+(\alpha-1)c\lambda ln\sum_{i=1}^{n}\frac{\left(\beta\left(e^{-\theta x}\right)^{c}\right)\left(\left(1-e^{-\theta x}\right)^{\beta c}\right)}{\left(1-(1-e^{-\theta x})^{\beta^2 c}\right)}-\alpha c\lambda ln\sum_{i=1}^{n}\frac{\left(\beta\left(e^{-\theta x}\right)^{c}\right)\left(\left(1-e^{-\theta x}\right)^{\beta c}\right)}{\left(1-(1-e^{-\theta x})^{\beta^2 c}\right)}$$

$$\frac{\partial^2 L}{\partial\beta\partial c}=n+(\alpha-1)\lambda ln\sum_{i=1}^{n}\frac{\left(\beta c\left(e^{-\theta x}\right)-1\right)\left(\beta\left(1-e^{-\theta x}\right)^{\beta c}\right)}{\left(1-(1-e^{-\theta x})^{\beta c}\right)}-\alpha\lambda ln\sum_{i=1}^{n}\frac{\left(\beta c\left(e^{-\theta x}\right)-1\right)\left(\beta\left(1-e^{-\theta x}\right)^{\beta c}\right)}{\left(1-(1-e^{-\theta x})^{\beta c}\right)}$$

$$\frac{\partial^2 L}{\partial\beta\partial\lambda}=n+(\alpha-1)\sum_{i=1}^{n}\frac{\left(\beta\left(e^{-\theta x}\right)^{c}\right)\left(\left(1-e^{-\theta x}\right)^{\beta c}\right)}{\left(1-(1-e^{-\theta x})^{\beta^2 c}\right)}-\alpha\sum_{i=1}^{n}\frac{\left(\beta\left(e^{-\theta x}\right)^{c}\right)\left(\left(1-e^{-\theta x}\right)^{\beta c}\right)}{\left(1-(1-e^{-\theta x})^{\beta^2 c}\right)}$$

$$\frac{\partial^2 L}{\partial\theta^2}=\frac{n}{\theta^2}+(\beta-1)\sum_{i=1}^{n}\frac{\left(\left(e^{-\theta x}\right)^{\beta c}\right)}{\left(1-(1-e^{-\theta x})^{\beta c}\right)^2}-\alpha\beta^2c^2\lambda^2\sum_{i=1}^{n}\frac{\left(\left(e^{-\theta x}\right)^{\beta c}\right)}{\left(1-(1-e^{-\theta x})^{\beta c}\right)^2}$$

$$\frac{\partial^2 L}{\partial c^2}=\frac{n}{c^2}+(\alpha-1)\beta\lambda ln\sum_{i=1}^{n}\frac{\left(\beta\left(e^{-\theta x}\right)^{2}\right)\left(\left(1-e^{-\theta x}\right)^{\beta c}\right)}{\left(1-(1-e^{-\theta x})^{\beta c^2}\right)}-\alpha\beta\lambda ln\sum_{i=1}^{n}\frac{\left(\beta\left(e^{-\theta x}\right)^{2}\right)\left(\left(1-e^{-\theta x}\right)^{\beta c}\right)}{\left(1-(1-e^{-\theta x})^{\beta c^2}\right)}$$

$$\frac{\partial^2 L}{\partial c\partial\lambda}=n+(\alpha-1)\beta\sum_{i=1}^{n}\frac{\left(e^{-\theta x}\right)^{\beta c}\left(1-e^{-\theta x}\right)^{\beta c}}{1-(1-e^{-\theta x})^{\beta c}}-\alpha\beta\sum_{i=1}^{n}\frac{\left(e^{-\theta x}\right)^{\beta c}\left(1-e^{-\theta x}\right)^{\beta c}}{1-(1-e^{-\theta x})^{\beta c}}$$

$$\frac{\partial^2 L}{\partial c \partial \theta} = n + \sum_{i=1}^{n} \frac{c\left(e^{-\theta x}\right)^{\beta}\left(1 - e^{-\theta x}\right)^{\beta c}}{1 - (1 - e^{-\theta x})^{\beta c}} - \sum_{i=1}^{n} \frac{c\left(e^{-\theta x}\right)^{\beta}\left(1 - e^{-\theta x}\right)^{\beta c}}{1 - (1 - e^{-\theta x})^{\beta c}}$$

$$\frac{\partial^2 L}{\partial \lambda^2} = \frac{n}{\lambda^2} \sum_{i=1}^{n} \frac{\left(1 - e^{-\theta x}\right)^{\beta c}}{1 - (1 - e^{-\theta x})^{\beta c}}$$

$$\frac{\partial^2 L}{\partial \lambda \partial \theta} = n + (\alpha - 1) \sum_{i=1}^{n} \frac{\left(1 - e^{-\theta x}\right)^{\beta c}}{1 - (1 - e^{-\theta x})^{\beta c}} - \alpha \sum_{i=1}^{n} \frac{\left(1 - e^{-\theta x}\right)^{\beta c}}{1 - (1 - e^{-\theta x})^{\beta c}}$$

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