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The Direct Simulation of Third Order Linear Problems on Single Step Block Method

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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ABSTRACT

In this article, the direct simulation of third order linear problems on single step block method has been proposed. In order to overcoming the setbacks in reduction method, direct method has been proposed using power series to reduce computational burden that occur in the reduction method. Numerical properties for the block method are established and the method developed is consistent, convergent and zero-stable. To validate the accuracy of the block method, certain numerical test problems were considered, the results shown that the accuracy of our method are more accurate over the existing method in literature.

Keywords: Direct method; computational burden; linear problems; reduction method; single step; simulation; third order.

1. INTRODUCTION

Mathematical models in the field of sciences and engineering are usually developed to understand the physical phenomena. These models are always resulted to differential equations. Ross [1] stated some of the problems that involved differential equations as 'the problem arising from

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determining the projectile motion, satellite, rocket or planet', 'the problem of how to determine the charge or current in an electric circuit', 'the study of chemical reactions' and 'the study of decomposition rate of radioactive substance or population growth rate'.

The problems mentioned above obey certain scientific laws that involve rates of change of one or more quantities. Mathematically, these rates of change can be expressed by derivatives. When the problems are converted to mathematical equations they will form differential equations.

In most application, higher order ordinary differential equations of the form

$$y^d = f\left(x, y(x), y'(x), \cdots, y^{d-1}(x), y^d(x)\right) \quad (1)$$

with conditions

$$y^s(a) = \eta_s, s = 0(1)d-1 \quad (2)$$

are first solved by a reduction to a system of first order ordinary differential equation of the form

$$y' = f(x,y), y(a) = y_0, a \le x \le b, x, y \in \Re \quad (3)$$

Then any suitable numerical methods would be used to solve the resulting equation. This approach is extensively discussed by scholars such as Spiegel [2], Lambert [3], Fatunla [4], Sarafyan [5] and Awoyemi [6]. It was noticed that this reduction process has a lots of setbacks such as difficulties in writing computer program for the method and computational burden which affects the accuracy of the method in terms of error. Therefore, in order to overcome these challenges, it will be appropriate and more efficient if direct method of solving (1) is employed as suggested by Dahlquist [7], Hall & Suleiman [8], Omar [9] and Kayode [10]. Some authors who adopt solving (1) directly are Adeyeye & Omar [11,12], Kuboye, Elusakin & Quadri [13], Raymond, Skwame & Adiku [14], Sabo, Althemai & Hamadina [15], Abdelrahim [16], Tumba, Skwame & Raymond [17].

The main aim of this article is to develop the third derivative block hybrid method for simulation of third order of linear problems using power series polynomial without reduction method.

2. MATHEMATICAL FORMULATION AND METHODOLOGY

This section described the development of third derivative block hybrid method using interpolation and collocation method for the direct solution of third order initial value problems of ordinary differential equation of the form (1). We consider one-step block method and obtained block method from the given expression in the form

$$y_{n+\xi} = \sum_{i=0}^{2} \frac{(\xi h)^i}{i!} y_n^{(i)} + \sum_{i=0}^{7} \left(\phi_{i\xi} f_{n+i} + \tau_{i\xi} g_{n+i}\right), \quad \xi = \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, 1 \quad (4)$$

obtain the first and second derivative schemes of the block method from

$$y_{n+\xi}^{(a)} = \sum_{i=0}^{2-a} \frac{(\xi h)^i}{i!} y_n^{(i+a)} + \sum_{i=0}^{7} \omega_{\xi i a} f_{n+i}, \, a = 1_{\left(\xi = \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, 1\right)}, \, a = 2_{\left(\xi = \frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, 1\right)} \quad (5)$$

$\phi_{\xi i} = A^{-1}B$ and $\omega_{\xi i a} = A^{-1}D$ where

$$A=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & \frac{h}{10} & \frac{h}{5} & \frac{3h}{10} & \frac{2h}{5} & \frac{h}{2} & \frac{3h}{5} & h \\ 0 & \frac{\left(\frac{h}{10}\right)^2}{2!} & \frac{\left(\frac{h}{5}\right)^2}{2!} & \frac{\left(\frac{3h}{10}\right)^2}{2!} & \frac{\left(\frac{2h}{5}\right)^2}{2!} & \frac{\left(\frac{h}{2}\right)^2}{2!} & \frac{\left(\frac{3h}{5}\right)^2}{2!} & \frac{(h)^2}{2!} \\ 0 & \frac{\left(\frac{h}{10}\right)^3}{3!} & \frac{\left(\frac{h}{5}\right)^3}{3!} & \frac{\left(\frac{3h}{10}\right)^3}{3!} & \frac{\left(\frac{2h}{5}\right)^3}{3!} & \frac{\left(\frac{h}{2}\right)^3}{3!} & \frac{\left(\frac{3h}{5}\right)^3}{3!} & \frac{(h)^3}{3!} \\ 0 & \frac{\left(\frac{h}{10}\right)^4}{4!} & \frac{\left(\frac{h}{5}\right)^4}{4!} & \frac{\left(\frac{3h}{10}\right)^4}{4!} & \frac{\left(\frac{2h}{5}\right)^4}{4!} & \frac{\left(\frac{h}{2}\right)^4}{4!} & \frac{\left(\frac{3h}{5}\right)^4}{4!} & \frac{(h)^4}{4!} \\ 0 & \frac{\left(\frac{h}{10}\right)^5}{5!} & \frac{\left(\frac{h}{5}\right)^5}{5!} & \frac{\left(\frac{3h}{10}\right)^5}{5!} & \frac{\left(\frac{2h}{5}\right)^5}{5!} & \frac{\left(\frac{h}{2}\right)^5}{5!} & \frac{\left(\frac{3h}{5}\right)^5}{5!} & \frac{(h)^5}{5!} \\ 0 & \frac{\left(\frac{h}{10}\right)^6}{6!} & \frac{\left(\frac{h}{5}\right)^6}{6!} & \frac{\left(\frac{3h}{10}\right)^6}{6!} & \frac{\left(\frac{2h}{5}\right)^6}{6!} & \frac{\left(\frac{h}{2}\right)^6}{6!} & \frac{\left(\frac{3h}{5}\right)^6}{6!} & \frac{(h)^6}{6!} \\ 0 & \frac{\left(\frac{h}{10}\right)^7}{7!} & \frac{\left(\frac{h}{5}\right)^7}{7!} & \frac{\left(\frac{3h}{10}\right)^7}{7!} & \frac{\left(\frac{2h}{5}\right)^7}{7!} & \frac{\left(\frac{h}{2}\right)^7}{7!} & \frac{\left(\frac{3h}{5}\right)^7}{7!} & \frac{(h)^7}{7!} \end{pmatrix}, B=\begin{pmatrix} \frac{(\xi h)^3}{3!} \\ \frac{(\xi h)^4}{4!} \\ \frac{(\xi h)^5}{5!} \\ \frac{(\xi h)^6}{6!} \\ \frac{(\xi h)^7}{7!} \\ \frac{(\xi h)^8}{8!} \\ \frac{(\xi h)^9}{9!} \\ \frac{(\xi h)^{10}}{10!} \end{pmatrix}, D=\begin{pmatrix} \frac{(\xi h)^{3-a}}{(3-a)!} \\ \frac{(\xi h)^{4-a}}{(4-a)!} \\ \frac{(\xi h)^{5-a}}{(5-a)!} \\ \frac{(\xi h)^{6-a}}{(6-a)!} \\ \frac{(\xi h)^{7-a}}{(7-a)!} \\ \frac{(\xi h)^{8-a}}{(8-a)!} \\ \frac{(\xi h)^{9-a}}{(9-a)!} \\ \frac{(\xi h)^{10-a}}{(10-a)!} \end{pmatrix}$$

The implementation of (4) and (5), can also be written in the following form

$$\left.\begin{aligned}
y_{n+\frac{1}{10}} &= y_n + \frac{h}{10}y'_n + \frac{\left(\frac{h}{10}\right)^2}{2!}y''_n + \left[\phi_{10}f_n + \phi_{11}f_{n+\frac{1}{10}} + \phi_{12}f_{n+\frac{1}{5}} + \phi_{13}f_{n+\frac{3}{10}} + \phi_{14}f_{n+\frac{2}{5}} + \phi_{15}f_{n+\frac{1}{2}} + \phi_{16}f_{n+\frac{3}{5}} + \phi_{17}f_{n+1}\right] \\
y_{n+\frac{1}{5}} &= y_n + \frac{h}{5}y'_n + \frac{\left(\frac{h}{5}\right)^2}{2!}y''_n + \left[\phi_{20}f_n + \phi_{21}f_{n+\frac{1}{10}} + \phi_{22}f_{n+\frac{1}{5}} + \phi_{23}f_{n+\frac{3}{10}} + \phi_{24}f_{n+\frac{2}{5}} + \phi_{25}f_{n+\frac{1}{2}} + \phi_{26}f_{n+\frac{3}{5}} + \phi_{27}f_{n+1}\right] \\
y_{n+\frac{3}{10}} &= y_n + \frac{3h}{10}y'_n + \frac{\left(\frac{3h}{10}\right)^2}{2!}y''_n + \left[\phi_{30}f_n + \phi_{31}f_{n+\frac{1}{10}} + \phi_{32}f_{n+\frac{1}{5}} + \phi_{33}f_{n+\frac{3}{10}} + \phi_{34}f_{n+\frac{2}{5}} + \phi_{35}f_{n+\frac{1}{2}} + \phi_{36}f_{n+\frac{3}{5}} + \phi_{37}f_{n+1}\right] \\
y_{n+\frac{2}{5}} &= y_n + \frac{2h}{5}y'_n + \frac{\left(\frac{2h}{5}\right)^2}{2!}y''_n + \left[\phi_{40}f_n + \phi_{41}f_{n+\frac{1}{10}} + \phi_{42}f_{n+\frac{1}{5}} + \phi_{43}f_{n+\frac{3}{10}} + \phi_{44}f_{n+\frac{2}{5}} + \phi_{45}f_{n+\frac{1}{2}} + \phi_{46}f_{n+\frac{3}{5}} + \phi_{47}f_{n+1}\right] \\
y_{n+\frac{1}{2}} &= y_n + \frac{h}{2}y'_n + \frac{\left(\frac{h}{2}\right)^2}{2!}y''_n + \left[\phi_{50}f_n + \phi_{51}f_{n+\frac{1}{10}} + \phi_{52}f_{n+\frac{1}{5}} + \phi_{53}f_{n+\frac{3}{10}} + \phi_{54}f_{n+\frac{2}{5}} + \phi_{55}f_{n+\frac{1}{2}} + \phi_{56}f_{n+\frac{3}{5}} + \phi_{57}f_{n+1}\right] \\
y_{n+\frac{3}{5}} &= y_n + \frac{3h}{5}y'_n + \frac{\left(\frac{3h}{5}\right)^2}{2!}y''_n + \left[\phi_{60}f_n + \phi_{61}f_{n+\frac{1}{10}} + \phi_{62}f_{n+\frac{1}{5}} + \phi_{63}f_{n+\frac{3}{10}} + \phi_{64}f_{n+\frac{2}{5}} + \phi_{65}f_{n+\frac{1}{2}} + \phi_{66}f_{n+\frac{3}{5}} + \phi_{67}f_{n+1}\right] \\
y_{n+1} &= y_n + hy'_n + \frac{(h)^2}{2!}y''_n + \left[\phi_{70}f_n + \phi_{71}f_{n+\frac{1}{10}} + \phi_{72}f_{n+\frac{1}{5}} + \phi_{73}f_{n+\frac{3}{10}} + \phi_{74}f_{n+\frac{2}{5}} + \phi_{75}f_{n+\frac{1}{2}} + \phi_{76}f_{n+\frac{3}{5}} + \phi_{77}f_{n+1}\right]
\end{aligned}\right\} \quad (6)$$

$$\left.\begin{aligned}
y'_{n+\frac{1}{10}} &= y'_n + \frac{h}{10}y''_n + \left[\omega_{100}f_n + \omega_{101}f_{n+\frac{1}{10}} + \omega_{102}f_{n+\frac{1}{5}} + \omega_{103}f_{n+\frac{3}{10}} + \omega_{104}f_{n+\frac{2}{5}} + \omega_{105}f_{n+\frac{1}{2}} + \omega_{106}f_{n+\frac{3}{5}} + \omega_{107}f_{n+1}\right] \\
y'_{n+\frac{1}{5}} &= y'_n + \frac{h}{5}y''_n + \left[\omega_{200}f_n + \omega_{201}f_{n+\frac{1}{10}} + \omega_{202}f_{n+\frac{1}{5}} + \omega_{203}f_{n+\frac{3}{10}} + \omega_{204}f_{n+\frac{2}{5}} + \omega_{205}f_{n+\frac{1}{2}} + \omega_{206}f_{n+\frac{3}{5}} + \omega_{207}f_{n+1}\right] \\
y'_{n+\frac{3}{10}} &= y'_n + \frac{3h}{10}y''_n + \left[\omega_{300}f_n + \omega_{301}f_{n+\frac{1}{10}} + \omega_{302}f_{n+\frac{1}{5}} + \omega_{303}f_{n+\frac{3}{10}} + \omega_{304}f_{n+\frac{2}{5}} + \omega_{305}f_{n+\frac{1}{2}} + \omega_{306}f_{n+\frac{3}{5}} + \omega_{307}f_{n+1}\right] \\
y'_{n+\frac{2}{5}} &= y'_n + \frac{2h}{5}y''_n + \left[\omega_{400}f_n + \omega_{401}f_{n+\frac{1}{10}} + \omega_{402}f_{n+\frac{1}{5}} + \omega_{403}f_{n+\frac{3}{10}} + \omega_{404}f_{n+\frac{2}{5}} + \omega_{405}f_{n+\frac{1}{2}} + \omega_{406}f_{n+\frac{3}{5}} + \omega_{407}f_{n+1}\right] \\
y'_{n+\frac{1}{2}} &= y'_n + \frac{h}{2}y''_n + \left[\omega_{500}f_n + \omega_{501}f_{n+\frac{1}{10}} + \omega_{502}f_{n+\frac{1}{5}} + \omega_{503}f_{n+\frac{3}{10}} + \omega_{504}f_{n+\frac{2}{5}} + \omega_{505}f_{n+\frac{1}{2}} + \omega_{506}f_{n+\frac{3}{5}} + \omega_{507}f_{n+1}\right] \\
y'_{n+\frac{3}{5}} &= y'_n + \frac{3h}{5}y''_n + \left[\omega_{600}f_n + \omega_{601}f_{n+\frac{1}{10}} + \omega_{602}f_{n+\frac{1}{5}} + \omega_{603}f_{n+\frac{3}{10}} + \omega_{604}f_{n+\frac{2}{5}} + \omega_{605}f_{n+\frac{1}{2}} + \omega_{606}f_{n+\frac{3}{5}} + \omega_{607}f_{n+1}\right] \\
y'_{n+1} &= y'_n + hy''_n + \left[\omega_{700}f_n + \omega_{701}f_{n+\frac{1}{10}} + \omega_{702}f_{n+\frac{1}{5}} + \omega_{703}f_{n+\frac{3}{10}} + \omega_{704}f_{n+\frac{2}{5}} + \omega_{705}f_{n+\frac{1}{2}} + \omega_{706}f_{n+\frac{3}{5}} + \omega_{707}f_{n+1}\right]
\end{aligned}\right\} \quad (7)$$

And

$$\left.\begin{aligned}
y''_{n+\frac{1}{10}} &= y''_n + \left[\omega_{1000}f_n + \omega_{1001}f_{n+\frac{1}{10}} + \omega_{1002}f_{n+\frac{1}{5}} + \omega_{1003}f_{n+\frac{3}{10}} + \omega_{1004}f_{n+\frac{2}{5}} + \omega_{1005}f_{n+\frac{1}{2}} + \omega_{1006}f_{n+\frac{3}{5}} + \omega_{1007}f_{n+1}\right] \\
y''_{n+\frac{1}{5}} &= y''_n + \left[\omega_{2000}f_n + \omega_{2001}f_{n+\frac{1}{10}} + \omega_{2002}f_{n+\frac{1}{5}} + \omega_{2003}f_{n+\frac{3}{10}} + \omega_{2004}f_{n+\frac{2}{5}} + \omega_{2005}f_{n+\frac{1}{2}} + \omega_{2006}f_{n+\frac{3}{5}} + \omega_{2007}f_{n+1}\right] \\
y''_{n+\frac{3}{10}} &= y''_n + \left[\omega_{3000}f_n + \omega_{3001}f_{n+\frac{1}{10}} + \omega_{3002}f_{n+\frac{1}{5}} + \omega_{3003}f_{n+\frac{3}{10}} + \omega_{3004}f_{n+\frac{2}{5}} + \omega_{3005}f_{n+\frac{1}{2}} + \omega_{3006}f_{n+\frac{3}{5}} + \omega_{3007}f_{n+1}\right] \\
y''_{n+\frac{2}{5}} &= y''_n + \left[\omega_{4000}f_n + \omega_{4001}f_{n+\frac{1}{10}} + \omega_{4002}f_{n+\frac{1}{5}} + \omega_{4003}f_{n+\frac{3}{10}} + \omega_{4004}f_{n+\frac{2}{5}} + \omega_{4005}f_{n+\frac{1}{2}} + \omega_{4006}f_{n+\frac{3}{5}} + \omega_{4007}f_{n+1}\right] \\
y''_{n+\frac{1}{2}} &= y''_n + \left[\omega_{5000}f_n + \omega_{5001}f_{n+\frac{1}{10}} + \omega_{5002}f_{n+\frac{1}{5}} + \omega_{5003}f_{n+\frac{3}{10}} + \omega_{5004}f_{n+\frac{2}{5}} + \omega_{5005}f_{n+\frac{1}{2}} + \omega_{5006}f_{n+\frac{3}{5}} + \omega_{5007}f_{n+1}\right] \\
y''_{n+\frac{3}{5}} &= y''_n + \left[\omega_{6000}f_n + \omega_{6001}f_{n+\frac{1}{10}} + \omega_{6002}f_{n+\frac{1}{5}} + \omega_{6003}f_{n+\frac{3}{10}} + \omega_{6004}f_{n+\frac{2}{5}} + \omega_{6005}f_{n+\frac{1}{2}} + \omega_{6006}f_{n+\frac{3}{5}} + \omega_{6007}f_{n+1}\right] \\
y''_{n+1} &= y''_n + \left[\omega_{7000}f_n + \omega_{7001}f_{n+\frac{1}{10}} + \omega_{7002}f_{n+\frac{1}{5}} + \omega_{7003}f_{n+\frac{3}{10}} + \omega_{7004}f_{n+\frac{2}{5}} + \omega_{7005}f_{n+\frac{1}{2}} + \omega_{7006}f_{n+\frac{3}{5}} + \omega_{7007}f_{n+1}\right]
\end{aligned}\right\} \quad (8)$$

To obtain the unknown coefficients ϕ, it is defined that $\phi_{\xi i} = A^{-1}B$ where A and are as given above. Therefore, using (6) with (4) and (5), we obtain

$$(\phi_{10},\phi_{11},\phi_{12},\phi_{13},\phi_{14},\phi_{15},\phi_{16},\phi_{17})=$$
$$\left(\frac{281833h^3}{3024000000},\frac{1457h^3}{9720000},-\frac{159931h^3}{967680000},\frac{319h^3}{2016000},-\frac{4583h^3}{45360000},\frac{28871h^3}{756000000},-\frac{127h^3}{19353600},\frac{4001h^3}{217728000000}\right)$$
$$(\phi_{20},\phi_{21},\phi_{22},\phi_{23},\phi_{24},\phi_{25},\phi_{26},\phi_{27})=$$
$$\left(\frac{67477h^3}{141750000},\frac{227h^3}{170100},-\frac{377h^3}{360000},\frac{719h^3}{708750},-\frac{3667h^3}{5670000},\frac{961h^3}{3937500},-\frac{19h^3}{453600},\frac{199h^3}{1701000000}\right)$$
$$(\phi_{30},\phi_{31},\phi_{32},\phi_{33},\phi_{34},\phi_{35},\phi_{36},\phi_{37})=$$
$$\left(\frac{32373h^3}{28000000},\frac{9h^3}{2240},-\frac{74439h^3}{35840000},\frac{2799h^3}{1120000},-\frac{1431h^3}{896000},\frac{16929h^3}{28000000},-\frac{1863h^3}{17920000},\frac{261h^3}{896000000}\right)$$
$$(\phi_{40},\phi_{41},\phi_{42},\phi_{43},\phi_{44},\phi_{45},\phi_{46},\phi_{47})=$$
$$\left(\frac{2101h^3}{984375},\frac{1744h^3}{212625},-\frac{131h^3}{47250},\frac{608h^3}{118125},-\frac{151h^3}{50625},\frac{3344h^3}{2953125},-\frac{23h^3}{118125},\frac{29h^3}{53156250}\right)$$
$$(\phi_{50},\phi_{51},\phi_{52},\phi_{53},\phi_{54},\phi_{55},\phi_{56},\phi_{57})=$$
$$\left(\frac{9899h^3}{2903040},\frac{3025h^3}{217728}h^3,-\frac{1625h^3}{516096},\frac{1375h^3}{145152},-\frac{625h^3}{145152},\frac{7h^3}{3840},-\frac{725h^3}{2322432},\frac{61h^3}{69672960}\right)$$
$$(\phi_{60},\phi_{61},\phi_{62},\phi_{63},\phi_{64},\phi_{65},\phi_{66},\phi_{67})=$$
$$\left(\frac{8721h^3}{1750000},\frac{369h^3}{17500},-\frac{891h^3}{280000},\frac{27h^3}{1750},-\frac{351h^3}{70000},\frac{1377h^3}{437500},-\frac{9h^3}{20000},\frac{9h^3}{7000000}\right)$$
$$(\phi_{70},\phi_{71},\phi_{72},\phi_{73},\phi_{74},\phi_{75},\phi_{76},\phi_{77})=$$
$$\left(\frac{299h^3}{15120},\frac{155h^3}{6804},\frac{1675h^3}{12096},-\frac{25h^3}{126},\frac{2525h^3}{9072},-\frac{617h^3}{3780},\frac{415h^3}{6048},\frac{179h^3}{544320}\right)$$

$$(\omega_{100},\omega_{101},\omega_{102},\omega_{103},\omega_{104},\omega_{105},\omega_{106},\omega_{107})=$$
$$\left(\frac{1049479h^2}{453600000},\frac{197687h^2}{38880000},-\frac{248029h^2}{48384000},\frac{251h^2}{51840},-\frac{13943h^2}{4536000},\frac{175109h^2}{151200000},-\frac{72067h^2}{362880000},\frac{6031h^2}{1088640000000}\right)$$
$$(\omega_{200},\omega_{201},\omega_{202},\omega_{203},\omega_{204},\omega_{205},\omega_{206},\omega_{207})=$$
$$\left(\frac{37697h^2}{7087500},\frac{20458h^2}{1063125},-\frac{1193h^2}{108000},\frac{824h^2}{70875},-\frac{17h^2}{2268},\frac{1678h^2}{590625},-\frac{2771h^2}{5670000},\frac{233h^2}{170100000}\right)$$
$$(\omega_{300},\omega_{301},\omega_{302},\omega_{303},\omega_{304},\omega_{305},\omega_{306},\omega_{307})=$$
$$\left(\frac{92859h^2}{11200000},\frac{38469h^2}{1120000},-\frac{783h^2}{89600},\frac{2151h^2}{112000},-\frac{5283h^2}{448000},\frac{24921h^2}{5600000},-\frac{1713h^2}{2240000},\frac{3h^2}{1400000}\right)$$
$$(\omega_{500},\omega_{501},\omega_{502},\omega_{503},\omega_{504},\omega_{505},\omega_{506},\omega_{507})=$$
$$\left(\frac{517h^2}{36288},\frac{28045h^2}{435456},-\frac{775h^2}{387072},\frac{3725h^2}{72576},-\frac{1475h^2}{145152},\frac{59h^2}{6912},-\frac{785h^2}{580608},\frac{13h^2}{3483648}\right)$$
$$(\omega_{600},\omega_{601},\omega_{602},\omega_{603},\omega_{604},\omega_{605},\omega_{606},\omega_{607})=$$
$$\left(\frac{753h^2}{43750},\frac{348h^2}{4375},-\frac{27h^2}{28000},\frac{12h^2}{175},-\frac{9h^2}{1750},\frac{432h^2}{21875},-\frac{9h^2}{100000},\frac{3h^2}{700000}\right)$$
$$(\omega_{700},\omega_{701},\omega_{702},\omega_{703},\omega_{704},\omega_{705},\omega_{706},\omega_{707})=$$
$$\left(\frac{215h^2}{2268},-\frac{610h^2}{1701},\frac{9925h^2}{6048},-\frac{1600h^2}{567},\frac{7325h^2}{2268},-\frac{374h^2}{189},\frac{6245h^2}{9072},\frac{293h^2}{54432}\right)$$

$$(\omega_{1000}, \omega_{1001}, \omega_{1002}, \omega_{1003}, \omega_{1004}, \omega_{1005}, \omega_{1006}, \omega_{1007}) =$$
$$\left(\frac{74423h}{2419200}, \frac{204961h}{1814400}, -\frac{98639h}{1075200}, \frac{3661h}{43200}, -\frac{128969h}{2419200}, \frac{1343h}{67200}, -\frac{16529h}{4838400}, \frac{11h}{1161216}\right)$$
$$(\omega_{2000}, \omega_{2001}, \omega_{2002}, \omega_{2003}, \omega_{2004}, \omega_{2005}, \omega_{2006}, \omega_{2007}) =$$
$$\left(\frac{1861h}{63000}, \frac{2171h}{14175}, -\frac{43h}{4200}, \frac{82h}{1575}, -\frac{1367h}{37800}, \frac{37h}{2625}, -\frac{31h}{12600}, \frac{h}{141750}\right)$$
$$(\omega_{3000}, \omega_{3001}, \omega_{3002}, \omega_{3003}, \omega_{3004}, \omega_{3005}, \omega_{3006}, \omega_{3007}) =$$
$$\left(\frac{2677h}{89600}, \frac{669h}{4480}, -\frac{13851h}{358400}, \frac{1313h}{11200}, -\frac{4491h}{89600}, \frac{81h}{4480}, -\frac{547h}{179200}, \frac{3h}{358400}\right)$$
$$(\omega_{4000}, \omega_{4001}, \omega_{4002}, \omega_{4003}, \omega_{4004}, \omega_{4005}, \omega_{4006}, \omega_{4007}) =$$
$$\left(\frac{701h}{23625}, \frac{2144h}{14175}, -\frac{31h}{1050}, \frac{832h}{4725}, -\frac{17h}{4725}, \frac{32h}{2625}, -\frac{11h}{4725}, \frac{h}{141750}\right)$$
$$(\omega_{5000}, \omega_{5001}, \omega_{5002}, \omega_{5003}, \omega_{5004}, \omega_{5005}, \omega_{5006}, \omega_{5007}) =$$
$$\left(\frac{965h}{32256}, \frac{10825h}{72567}, \frac{1625h}{43008}, \frac{625h}{4032}, \frac{7375h}{96768}, \frac{151h}{2688}, -\frac{275h}{64512}, \frac{11h}{1161216}\right)$$
$$(\omega_{6000}, \omega_{6001}, \omega_{6002}, \omega_{6003}, \omega_{6004}, \omega_{6005}, \omega_{6006}, \omega_{6007}) =$$
$$\left(\frac{41h}{1400}, \frac{27h}{175}, \frac{27h}{1400}, \frac{34h}{175}, \frac{27h}{1400}, \frac{27h}{175}, \frac{41h}{1400}, 0\right)$$
$$(\omega_{7000}, \omega_{7001}, \omega_{7002}, \omega_{7003}, \omega_{7004}, \omega_{7005}, \omega_{7006}, \omega_{7007}) =$$
$$\left(\frac{827h}{1512}, -\frac{2125h}{567}, \frac{2125h}{168}, -\frac{4250h}{189}, \frac{36625h}{1512}, -\frac{311h}{21}, \frac{6875h}{1512}, \frac{89h}{1134}\right)$$

3. INVESTIGATING THE PROPERTIES OF THE BLOCK METHOD

We shall analyze the basic properties of the method derived, which include order, error constant, consistency, zero-stable and convergence.

3.1 Order and Error Constant of the Block Method

According to Lambert [3] and Tumba, Skwame & Raymond [17], the order and error constant of the block method

$$y^{(m)}(x_n + ah) = y^{(m)}(x_n) + ahy^{(m+1)}(x_n) + \frac{(ah)^2}{2!} y^{(m+2)}(x_n) + \frac{(ah)^3}{3!} y^{(m+3)}(x_n) + \cdots \quad (9)$$

where

$$y^{(m)}x_n = \left.\frac{d^m y}{dx^m}\right|_{x=x_n}, m = 1, 2, \cdots$$

Using the linear operator

$$L[y(x);h] = \sum_{j=0}^{1} \alpha_j y_{n+j} - \sum_{j=0}^{1} \beta_j f_{n+j} + \sum_{j=0}^{1} \gamma_j f'_{n+j} \quad (10)$$

Expanded (10) using Taylor series expansions about x_n and comparing the coefficient h and the method is said to be of order p if $C_0 = C_1 = \cdots = C_{p+1} = 0, C_{p+2} = 0, C_{p+3} \neq 0$ and C_{p+3} is the error constant.

Therefore the order and error constant of third derivative block method are $p = [6, 6, 6, 6, 6, 6, 6]$ and

$$C_{p+3} = [\text{-}1.3619\text{e-}11, \text{-}1.0520\text{e-}11, \text{-}1.2003\text{e-}11, \text{-}1.0723\text{e-}11, -1.2702\text{e-}11, \text{-}6.4286\text{e-}12, \text{-}8.0743\text{e-}09]^T$$

3.2 Consistency

The block method is said to be consistent if the order is greater than or equal to one i.e. $p \geq 1$.

3.3 Zero Stability

The block method is said to be zero-stable, if the roots $z_s, s = 1, 2, \cdots, k$ of the first characteristics polynomial $\rho(z)$ defined by $\rho(z) = \det\left(zA^{(0)} - E\right)$ satisfies $|z_s| \le 1$ and every root satisfies $|z_s| = 1$ have multiplicity not exceeding the order of the differential equation, Omar [18]. To analyze the block method for zero stability, the roots of the first characteristic polynomial

$$\rho(z) = \left| zI_7 - \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right| \qquad (11)$$

Solving for z in (11) gives $\rho(z) = z^6 - z^5$.

3.4 Convergence

The block method is to be convergent if it is consistent and zero-stable. Hence the block method is convergent, Omar [18].

4. NUMERICAL IMPLEMENTATION OF THE PROBLEMS

The accuracy and convergence of the block method will be studied using some highly stiff third order linear problems, textual form and graphically shown.

Problem one: Consider the third order linear problem

$$y''' + 4y' - x = 0,\ y(0) = y'(0) = 0,\ y''(0) = -1,\ h = 0.1$$

with the exact solution given by

$$y(x) = \frac{3}{16}(1 - \cos 2x) + \frac{x^2}{8}$$

Problem two: Consider the third order linear problem

$$y'''(g) = 3\cos(x), \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 2$$

with the exact solution: $y(x) = x^2 - 3\sin(x) + 3x + 1$

Problem three: Consider the third order linear problem

$$y'''(g) = \exp(x), \quad y(0) = 3,\ y'(0) = 1,\ y''(0) = 5,$$

with the exact solution: $y(x) = 2 + 2x^2 + \exp(x)$

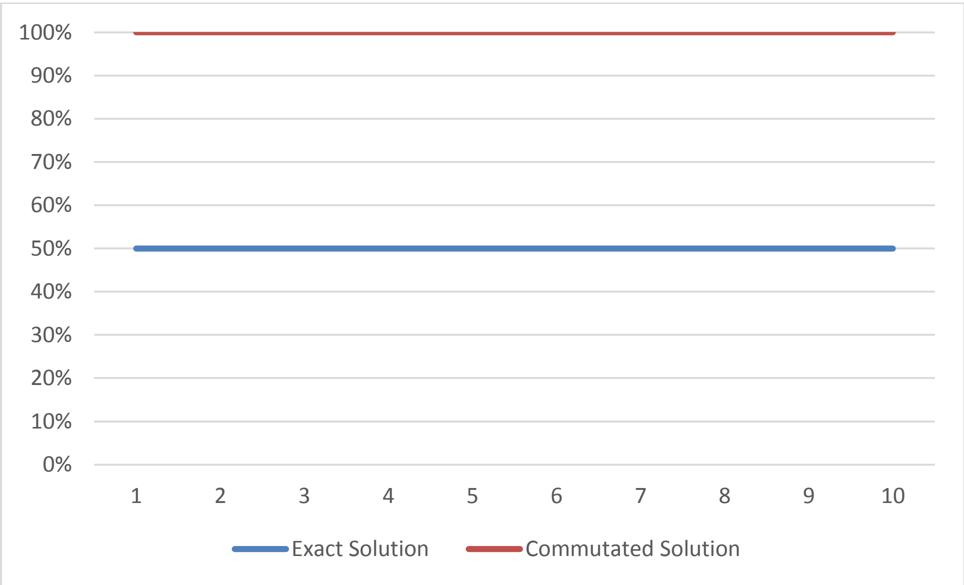

Fig. 1. The graphical solution of problem one

Table 1. Showing the comparison of error for problem one

x-value	Exact Solution	Commutated Solution	Error in new Method	Error in Adoghe & Omole [19]	Error in Adeyeye & Omar [19]	Error in Aigbiremhon & Omole [19]
0.1	0.00498751665476719416	0.00498751665476719414	2.0000e-20	1.7896e-17	0.2304e-14	2.8818e-09
0.2	0.01980106362445904698	0.01980106362445904517	1.8100e-18	5.7085e-13	0.1658e-13	3.2893e-08
0.3	0.04399957220443531927	0.04399957220443530750	1.1770e-17	2.7015e-12	0.4850e-13	1.1954e-07
0.4	0.07686749199740648358	0.07686749199740644174	4.1840e-17	7.3797e-12	0.1147e-12	2.8709e-07
0.5	0.11744331764972380299	0.11744331764972369448	1.0851e-16	1.5592e-11	0.2425e-12	5.5398e-07
0.6	0.16455792103562370419	0.16455792103562347275	2.3144e-16	2.8358e-11	0.4436e-12	9.2975e-07
0.7	0.21688116070620482401	0.21688116070620439221	4.3180e-16	4.6545e-11	0.7467e-12	1.4149e-06
0.8	0.27297491043149163616	0.27297491043149090574	7.3042e-16	7.1128e-11	0.1183e-11	1.9995e-06
0.9	0.33135039275495382287	0.33135039275495267713	1.1457e-15	1.0308e-10	0.1753e-11	2.6636e-06
1.0	0.39052753185258919756	0.39052753185258750565	1.6919e-15	1.4336e-10	0.2481e-11	3.3776e-06

Source [Adoghe & Omole [19]. Adeyeye & Omar [11] and Aigbiremhon & Omole [20]]

Table 2. Showing the comparison of error for problem two

x-value	Exact Result	Approximate Result	Error in our Method	Error in Taparki, Gurah & Simon [21]	Error in Skwame, et. al. [22]
0.1	1.01049975005951554310	1.01049975005951554310	0.0000	2.4800e-07	1.9700e-16
0.2	1.04399200761481635360	1.04399200761481635360	0.0000	7.3740e-06	1.2639e-15
0.3	1.10343938001598127470	1.10343938001598127530	6.0000e-19	6.0542e-05	4.0627e-15
0.4	1.19174497307404852500	1.19174497307404852670	1.7000e-18	2.5479e-04	9.4370e-15
0.5	1.31172338418739099920	1.31172338418739100290	3.7000e-18	7.7602e-04	1.8205e-14
0.6	1.46607257981489392840	1.46607257981489393520	6.8000e-18	1.9261e-03	3.1152e-14
0.7	1.65734693828692683900	1.65734693828692685030	1.1300e-17	4.1505e-03	4.9021e-14
0.8	1.88793172730143171510	1.88793172730143173240	1.7300e-17	8.3637e-03	7.2504e-14
0.9	2.16001927111754983460	2.16001927111754985950	2.4900e-17	1.0224e-13	1.0224e-13
1.0	2.47558704557631048000	2.47558704557631051450	3.4500e-17	1.3880e-13	1.3880e-13

Source: [Skwame, et. al. [22] and Taparki, Gurah & Simon [21]]

Table 3. Showing the comparison of error for problem three

x-value	Exact Result	approximate Result	Error in our Method	Error in Kuboye & Omar [23]	Error in Areo & Omojola [24]	Error in Adeyeye & Omar, [25]
0.1	3.12517091807564762480	3.12517091807564762480	0.0000	2.2205e-15	2.6645e-15	6.3427e-13
0.2	3.30140275816016983390	3.30140275816016983390	0.0000	1.4211e-14	4.4409e-16	2.3288e-12
0.3	3.52985880757600310400	3.52985880757600310430	3.0000e-19	3.6415e-14	3.1086e-15	5.4435e-12
0.4	3.81182469764127031780	3.81182469764127031860	8.0000e-19	6.8390e-14	6.6613e-15	9.8532e-12
0.5	4.14872127070012814680	4.14872127070012814840	1.6000e-18	1.0925e-13	9.7697e-15	1.5997e-11
0.6	4.54211880039050897490	4.54211880039050897760	2.7000e-18	1.6076e-13	2.0428e-14	2.3722e-11
0.7	4.99375270747047652160	4.99375270747047652610	4.5000e-18	2.2116e-13	2.1316e-14	3.3568e-11
0.8	5.50554092849246760460	5.50554092849246761160	7.0000e-18	2.9221e-13	1.8652e-14	4.5344e-11
0.9	6.07960311115694966380	6.07960311115694967450	1.0700e-17	3.6948e-13	2.2205e-14	5.9708e-11
1.0	6.71828182845904523540	6.71828182845904525080	1.5400e-17	4.6718e-13	2.1316e-14	7.6432e-11

Source: [Kuboye & Omar [23], Areo & Omojola [24] and Adeyeye & Omar [25]]

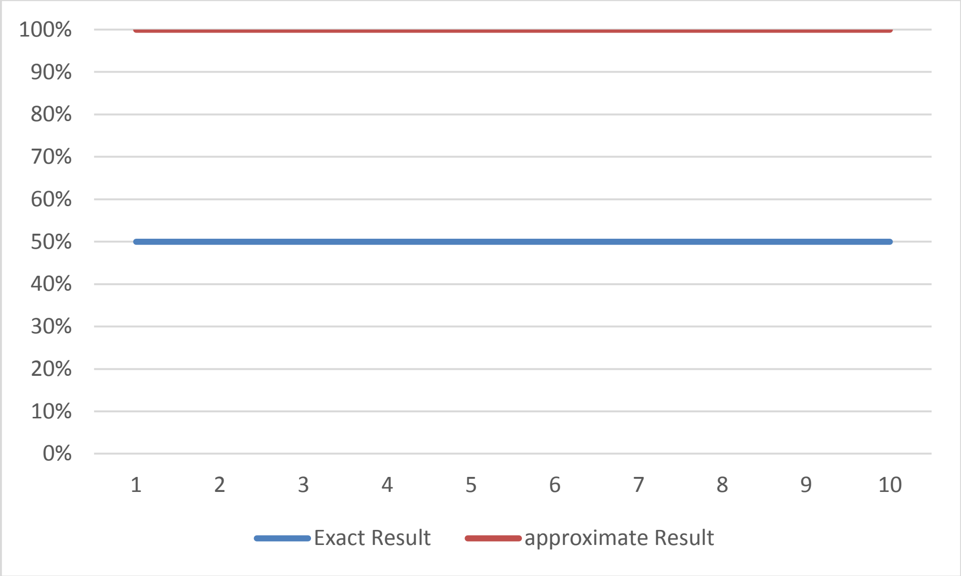

Fig. 2. The graphical solution of problem two

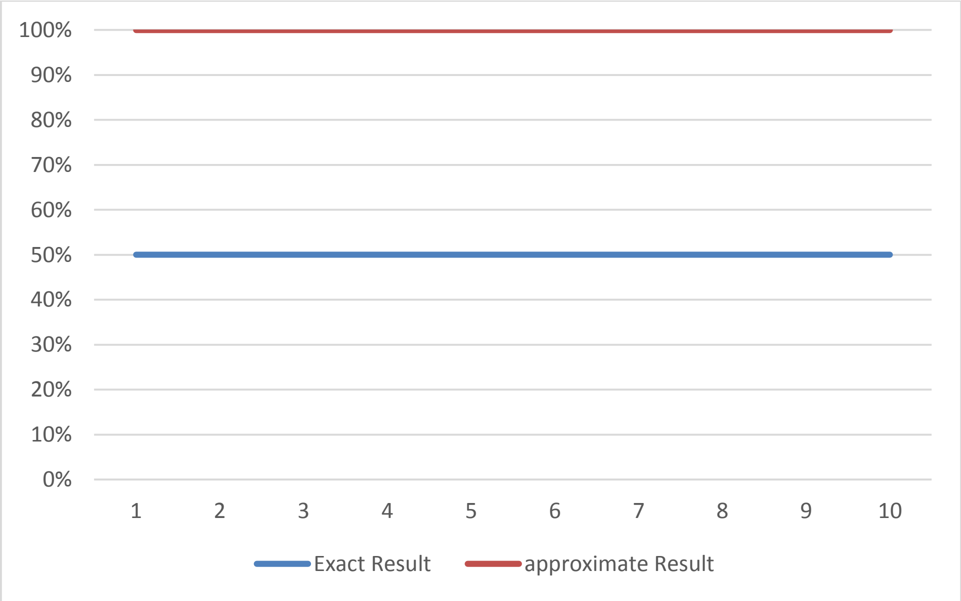

Fig. 3. The graphical solution of problem three

5. SUMMARY AND CONCLUSION

Conventionally, initial value problems of higher order ordinary differential equations are solved by first reducing the equations to their equivalent systems of first order ordinary differential equations. Then, suitable existing numerical methods for first order ordinary differential equations will be employed to solve the resulting equations. However, this reduction process involve a computational fact such as difficulties in writing computer program for the method and computational burden which affects the accuracy of the method in terms of error. Now, in other to overcome the setbacks, this article, proposed the direct simulation of third order initial value problems using power series. Numerical properties for the block method are established and the method developed is consistent, convergent and zero-stable. To validate the superiority of the block method, certain numerical problems were considered, the results shown the accuracy of the method over the existing method in literature.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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