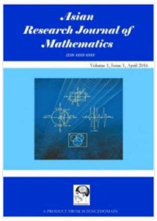

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A Genetic Algorithm with Semi-Greedy Heuristic Construction Phase for Multiple Fuzzy k-cardinality Assignment Problem with HOWA Approach

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Authors' contributions

This work was carried out in collaboration between both authors. Authors AM and BTT designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Authors AM and BTT managed the analyses of the study. Both authors read and approved the final manuscript.

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Abstract

The assignment problem is one of the well-known combinatorial optimization problems. It consists of finding a maximum or a minimum weight matching in a weighted bigraph. k-cardinality assignment problem is a special case of the assignment problem with side constraints. The scope of this study is to be able to suggest kind of group assignment problem with side constraint. The aim of the study is to create groups of workers in order to minimize the cost of the assignment.

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In that problem, costs of workers are stated as fuzzy numbers. Also; with this model, evaluation criteria for every group could be different from each other. We make that happen by employing HOWA (Heavy Ordered Weighted Averaging) aggregation operator. Using of HOWA in the objective function of the model transforms the model into a fuzzy non-linear programming model. We implement our model to "gap12" data from OR-Library. We solve this model employing both Genetic Algorithm in which is constructing the initial population by a semi-greedy heuristic, along with Parametric Programming. We also develop a user friendly interface that reports findings of the model to us.

Keywords: k-cardinality assignment; fuzzy costs; HOWA (Heavy Ordered Weighted Averaging); aggregation operator; genetic algorithm.

2010 Mathematics Subject Classification: 53C25; 83C05; 57N16

1 Introduction

The assignment problem, which is a common problem in the real world and usually met by decision makers, is one of the well-known combinatorial optimization problems. It may be evaluated as finding minimum (maximum) weight matching in a weighted bigraph. In the classical assignment problem, the aim is to find an assignment, or matching, between the elements of two (or more) sets, such that the total cost of all matched pairs (tuples) is minimized. Depending on the particular structure of the sets being matched, the form of the cost function, the matching rule, and so on, the assignment problems are categorized into linear, quadratic [1], bottleneck [2], multidimensional, etc. The assignment problems can be stated in a variety of forms, including mathematical programming, combinatorial, or graph-theoretic formulations, and constitute one of the most important and fundamental objects in the areas of computer science, operations research, and discrete mathematics [3]. That's why a lot of models and algorithms in a deterministic environment have existed.Even though most popular and widely used algorithm for the assignment problem is the Kuhns Hungarian method [4], various solution methods were proposed for the different type of the deterministic assignment problem [5, 6, 7].

However, in the application, because of the absence of historical data or other uncertain factors, the profit, the material cost and the consumed time by a worker on a task are not fixed. Therefore, it is not proper to utilize deterministic models and algorithms in these situations. In recent years, many researchers began to investigate this kind of problem under the uncertain environment [8]. Some of them employed the fuzzy theory to deal with nondeterministic factors for the assignment problem [9, 10, 11, 12, 13, 14]. Zadeh [15] initialized the fuzzy set theory in 1965. According to his paper, fuzzy logic could be used as a suitable approach for modeling uncertainty of various problems.Lin et.al. [9] investigated fuzzy assignment problem concerning sensitivity point of view. They modified well-known labeling algorithm denoted as LW [14] to obtain practical sensitivity ranges. Fachao et. al. [10] proposed the synthesizing effect function which was described based on $I_L$metric. They transformed the fuzzy assignment problem into a crisp 0-1 integer programming problem. Then, they solved the transformed problem by employing Genetic Algorithm. Pandian and Kavitha [11] constructed upper and lower crisp assignment problems for given fuzzy assignment problem. Based on the optimal solutions of these crisp problems, they obtained the optimal solution to the fuzzy assignment problem by employing the method called parallel moving. Gani and Mohammed [12] proposed a new ranking method for fuzzy numbers and employed the method to transform fuzzy assignment problem into a crisp one. Then, they solved the crisp problem by utilizing the Simplex method. Lin [13] considered team and manager performances to evaluate the fuzzy assignment problem. Based on the min-max criterion, Lin constructed a nonlinear general fuzzy assignment problem. Then Lin transformed the problem into a fractional programming problem and solved the problem by employing both together Simplex algorithm and trade-off approach. Dey et.al. [16] used an interval type-2 fuzzy sets to represent the arc lengths of a fuzzy graph for fuzzy shortest path problem. They called this problem an interval type-2 fuzzy shortest path problem. Also, they proposed an algorithm for it. Dey et.al. [17] used a fuzzy graph model to represent a traffic network of a city and discussed a method to find the different types of accidental zones in traffic flows. Dey et.al. [18] proposed a fuzzy graph model to represent the examination scheduling problem of a university and introduced a genetic algorithm based method to find the robust solution of the scheduling problem that remains feasible and optimal or close to optimal for all scenarios of the input data. Dey et.al.[19] introduced an algorithm to find the complement of any fuzzy graph with $O(n^2)$ time and also coloring this complement fuzzy graph using α cut. Dey. et.al [20] concentrated on a minimum spanning tree problem on a graph, in which either a trapezoidal fuzzy number or triangular fuzzy number, instead of a real number, is assigned to each arc length.

Fuzziness is not the unique way to model uncertainty in general. So, while modeling assignment problems including uncertainty, various scientists applied different ways of dealing with uncertainty [21, 22, 23, 24]. Mukherjee and Basu [21] constructed a method to solve intuitionistic fuzzy assignment problem with and without restrictions. They designed the method in two phases: in the first part, they obtained decision matrix with respect to restrictions of the problem. In the second part, they solved intuitionistic fuzzy assignment problem by calculating similarity between the decision matrix and pre-determined idealized solutions. Zhang and Peng [25] modeled profit of assignment problem by utilizing uncertain variable concept. They proposed the concept of α-optimal assignment for obtaining the solution of uncertain assignment problem. Gani et. al. [23] evaluated bottleneck assignment problem with intuitionistic fuzzy completion time. They also proposed an algorithm to solve the mentioned problem. Singh employed dual hesitant fuzzy sets to model assignment problem with restrictions. In order to obtain optimal solution for the problem, Singh [24] developed two algorithms that are based on the similarity which is also proposed by Singh.

k-cardinality assignment problem is a special case of the assignment problem with side constraint. The most general form of the problem is as follows: Choosing k workers and k tasks from m workers and n tasks such that the total weight is minimized (maximized) after all the selected jobs are completed by selected workers where, $k \leq \min\{m, n\}$ [25, 26]. The k-cardinality assignment problem has a lot of potential applications in real life and has direct applications in assigning workers to machines when there are multiple alternatives and only a subset of workers and machines is required [27]. Moreover, the problem can arise as a sub problem in the solution of more complex problems with the aid of k-cardinality assignment problem mentality such as, Satellite-Switched Time-Division Multiple Access time slot assignment problem [28].In order to solve k-cardinality assignment problem by employing any classical assignment problem algorithm, Volgenant [29] proposed a transformation. DellAmico et. al. [28] considered a special case of k-cardinality assignment problem in which some of the costs are infinite. For the case, they provided an algorithm and related computer code. Feng and Yang [8] proposed a fuzzy k-cardinality assignment problem with two objective functions which one of them is for total consumed time and the other is for total profit. They modelled profits and costs with fuzzy variables. For the solution of the problem, they constructed an algorithm based on goal programming and heuristic approaches. Zhang and Peng [25] also proposed a multi-objective k-cardinality problem with uncertain parameters. They evaluated three objectives (profit, cost and time). They also utilized goal programming approach.

In this paper, we shall consider a multiple fuzzy k-cardinality assignment problem with HOWA.We modelled the case that we would like to construct L groups for which the numbers of workers in each group are denoted as k_l in order to have $\sum k_l$ tasks done. Moreover, we choose $\sum k_l$ tasks and $\sum k_l$ workers from n tasks and m workers so that the total fuzzy cost of workers is minimized where $\sum k_l \leq \min\{m, n\}$. While evaluating the model, we gave flexibility to decision-maker by employing HOWA. HOWA is an extension of the OWA operator that allows the weighting vector to sum up to the number of item in a group. Using this exception, decision-maker can consider total cost of each group different from each other. Actually; this differentiation exposes some information concerning decision making attitude of the decision maker, such as being optimistic, pessimistic, or decision natural [30]. The main characteristic of this operator is that it provides a parameterized family of aggregation operators that includes among others, the minimum, the OWA operator and the total operator. As we can see, adjusting weight vector allows this operator to range between the OWA operator and the total operator [31]. Evaluating general performance of each group by utilizing HOWA makes our problem non-linear. To obtain the solution of our problem, we used Genetic Algorithm of which population generation phase, we employed Greedy Randomized Adaptive Search Procedure.

This paper is organized as follows. After introducing explained definition of multiple fuzzy k-cardinality assignment problem in Section 2, we construct the fuzzy non-linear programming model for the problem in Section 3. Genetic Algorithm with Semi-Greedy Heuristic Construction Phase (GASGCP) is described in Section 4. A numerical example is presented to show the application in Section 5.Finally, conclusion is given in Section 6.

2 Definition of Multiple Fuzzy k-cardinality Assignment Problem

The purpose of the k-cardinality assignment problem ,in general, is to choose k tasks and k workers from n tasks and m workers so that the total cost of workers is minimized where $k \leq \min\{m, n\}$. Moreover, a selected worker must take over just one of the selected k tasks. This problem may be evaluated as a problem of teaming up.

In this study, we assume that a decision-maker would like to build more than one team. To model the assumption, we add extra side constraints into classical k-cardinality assignment problem. We allow decision-maker to build more than one team of which numbers of members may be different for every team. The number of side constraints is equal to the number of teams.

Furthermore, decision-maker would like to measure general performance of each team with different scale. To realize the assumption, we exploit HOWA operators. With different weight vectors, HOWA turns into different aggregation operator. HOWA aggregation operator of dimension n is mapping $H : R^n \to R$ such that

$$HOWA(a_1, ..., a_n) = W^T B = \sum_{j=1}^{n} w_j a_{index(j)}$$

in which B is the ordered argument vector and W is a weighting vector such that
(i) $0 \leq w_j \leq 1$ and (ii) $1 \leq \sum_{j=1}^{n} w_j \leq n$
As we can see, if $\sum_{j=1}^{n} w_j = 1$ then, we get the usual weighted average and if $\sum_{j=1}^{n} w_j = n$, we get the total operator [32].

In this definition, while, each of the weights is restricted to lie in the unit interval, the sum of the weights is allowed to be in $[1, n]$. Using these properties, the operator can cover wide range of aggregation operators. In our model, while we utilize HOWA operator, we allow decision-maker to determine different weight vector for different teams. So, decision-maker is able to measure performance of each team with different scale. Let's say, for one team, decision-maker would like to use "total cost" as a scale. For this case, W is such that $w_j = 1$ for all j. For another team,

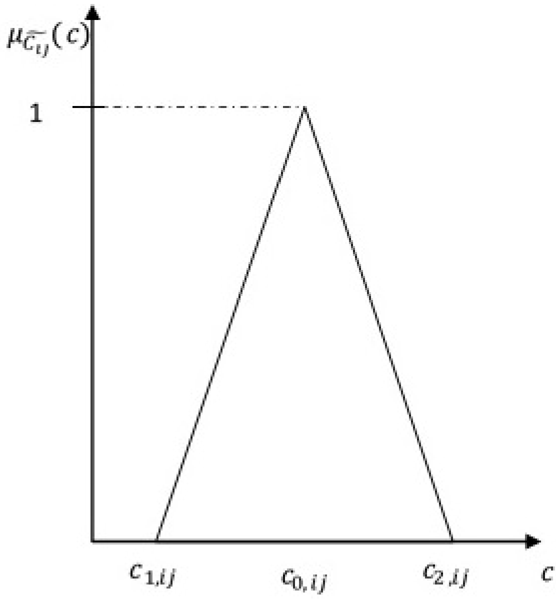


Fig. 1. Fuzzy membership function of cost for worker i's being assigned to task j

decision-maker, let's say, would like to employ "maximum cost in the team" as a scale. For this case, W is such that $\sum_{j=1}^{n} w_j = 1$, $w_1 = 1$ and $w_j = 0$ all $j \neq 1$.

To make the concerning problem more realistic, $\tilde{c}_{ij}$ are supposed to be fuzzy variables because, in some cases, knowing the concrete values of $\tilde{c}_{ij}$ may be impossible before tasks are completed. For the sake of simplicity, we use triangular fuzzy numbers (Fig. 1) to model fuzzy costs.

The membership function $\mu_{\widetilde{C_{ij}}}(c)$ is defined as

$$\mu_{\widetilde{C_{ij}}}(c) = \begin{cases} 0, & c \leq c_{1,ij} \\ \frac{c - c_{1,ij}}{c_{0,ij} - c_{1,ij}}, & c_{1,ij} \leq c \leq c_{0,ij} \\ \frac{c_{2,ij} - c}{c_{2,ij} - c_{0,ij}}, & c_{0,ij} \leq c \leq c_{2,ij} \\ 0, & c \geq c_{2,ij} \end{cases} \tag{2.1}$$

So far, we mentioned and explained main ideas and assumptions lying underneath our model. This model is a non-linear fuzzy programming problem model. To be able to handle fuzziness in the model, we employ parametric programming approach.

3 Fuzzy Non-Linear Programming Model

The model mentioned in the study is different from ordinary k-cardinality assignment problem in two ways: (i) Elements of cost matrix are fuzzy numbers. (ii) Using the model, decision-makers may realize an assignment consisting of more than one team(k_1, $k_2, \ldots,$ k_g). Moreover, decision-makers would like to evaluate different teams with different decision-making strategies. The model also gives opportunity to model this preference. We suppose that every worker can handle every task in return for some payment. These costs are modeled by fuzzy numbers. The notation of the model including index sets which are consisted by n workers and m tasks, fuzzy input parameters, and decision variables are as follows.

Index Sets

$i \in I$ Worker
$j \in J$ Task
$l \in L$ Group

Input Parameters

$\tilde{c}_{ij}$ Fuzzy cost of worker i 's performing task j

Decision Variables

$x_{ij} \in \{0, 1\} = 1$ worker i assigned to task j

Fuzzy Non-Linear Mathematical Model

$$z = \min\{\widetilde{HOWA}_1\,(\tilde{c}_{ij}x_{ij}) \oplus \cdots \oplus \widetilde{HOWA}_g\,(\tilde{c}_{ij}x_{ij})\} \tag{3.1}$$

$$\sum_J x_{ij} \leq 1 \quad \forall i \tag{3.2}$$

$$\sum_I x_{ij} \leq 1 \quad \forall j \tag{3.3}$$

$$\sum_I \sum_J x_{ij} = k_l \quad \forall l \tag{3.4}$$

$$\sum_L k_l \leq \min(m, n) \tag{3.5}$$

$$x_{ij} \in \{0, 1\} \quad \forall i \;\; \forall j \tag{3.6}$$

In the objective of the model, we aim to minimize the fuzzy total cost of assignments of workers to tasks. The constraint (3.2) ensures that the required number of workers are assigned to tasks. The inequality (3.3) provides that required number of tasks is covered by workers. The equality (3.4) ensures that all of the tasks in each team are matched to workers. The inequality (3.5) satisfies that the sum of the number of assigned tasks is allowed to be between 1 and $\min(m, n)$. Finally, (3.6) denotes integrality constraints.

There are several methods for solving fuzzy programming problems. Some of these methods can be found in [33, 34, 35, 36, 37, 38, 39]. In this study, the constraints of our model are linear. Actually, if we had not had HOWA operators in the objective function, our model would have turned into fuzzy linear programming problem. Fuzzy linear programming problems are generally solved by LP algorithms after transforming the fuzzy problem into crisp linear programming problem at different α cut levels. Also, we will follow a similar approach. We will calculate lower and upper bounds of fuzzy costs at different α cut levels. For each α cut level, we will replace first crisp lower bounds and then crisp upper bounds with fuzzy costs. Therefore, we will solve two non-linear programming problems for each α cut level. This turns our original problem into parametric non-linear programming model. The model is as follows.

$$z = \min\{HOWA_1\,(c_{\alpha ij}x_{ij}) + \cdots + HOWA_g\,(c_{\alpha ij}x_{ij})\} \tag{3.7}$$

$$\sum_J x_{ij} \leq 1 \quad \forall i \tag{3.8}$$

$$\sum_I x_{ij} \leq 1 \quad \forall j \tag{3.9}$$

$$\sum_I \sum_J x_{ij} = k_l \quad \forall l \tag{3.10}$$

$$\sum_L k_l \leq \min(m, n) \tag{3.11}$$

$$x_{ij} \in \{0, 1\} \quad \forall i \ \ \forall j \tag{3.12}$$

All constraints, are identified by 3.8,3.9,3.10,3.11,3.12, have the same meaning previous constraints. We employ Genetic Algorithm (GA) to solve these problems.

4 Genetic Algorithm with Semi-Greedy Heuristic Construction Phase

We utilize (GA) method to solve the parametric programming model. GA has been recently applied for solving various mathematical programing problems. GA is a kind of solution generator, in addition to solution selector. GA idea belongs to J. Holland. This idea was proposed by studies of cellular automata, conducted by Holland and his student at the University of Michigan [40].

In general, GA begins to generate a set of n randomly selected states, called the population. Each state, that is called individual, is represented a string of 0s and 1s, but other encodings are also possible according to problem type. In our problem, 0-1 type encoding is employed because it is best suitable alternative. In used method to encode chromosomes of the problem, each worker has a boolean variable to indicate its assignment. For example, in the n workers and m tasks problem, the number of genes in the chromosomes is $n \times m$. The fitness function is calculated for the production of the next generation of states. For our problem, objective function is chosen as the fitness function. The fitness function rates each state. In our problem, state refers to an assignment. A fitness score is calculated by fitness function for all states, the probabilities of selection for reproducing are generated by fitness scores. Then the percentages are shown for selecting next generation. In selection step, a pair is selected randomly for reproduction, according to probabilities which are calculated by fitness scores. When all pairs are mated, selection step finish. After selection, a crossover point is chosen randomly for each pair. The offspring is created by crossing over the parent from crossover point. When parent states are quite different, a state, which is produced by crossing over, is different from either parent states. So crossover frequently takes large step in the state space early in the process but smaller step later. It's about most states are similar when later in the process. Finally, in the last step, each state is modified by random mutation with a probability. Mutation is actualized by one digit's transformation. One last step which is called repair is performed as needed. In this step, if a state doesn't satisfie constraints then it is repaired. Repairs are performed as a worker is assigned to tasks is replaced with random one until state satisfies constraints. After all, steps are realized, best solution is kept in memory. This loop proceeds until termination, that is the criterion decides whether to continue searching or stop the search, is enabled [41, 42]. In our implementation of GA, termination condition is, fixed number of generations reached.

The performance of a GA is often depended to the quality of the initial population which is affected by both the average fitness of individuals and the diversity. We need balanced diversity which prevents early converge to a local optimal solution if its value is high, and high average fitness for better final individuals.

```
procedure construct(g(.),β,x);
x = ∅;
Initialize candidate set C;
while C ≠ ∅ do
    read current;
    s̲ = min{g(t) | t ∈ C};
    s̄ = max{g(t) | t ∈ C};
    RCL = {s ∈ C | g(s) ≤ s̲ + β(s̄ − s̲)};
    Select s, at random, from the RCL;
    x = x ∪ {s};
    Update candidate set C;
end
```

Algorithm 1: Construction phase

```
procedure GASGCP(c̃, H, β,k);
S̃ = ∅;
for each α cut of c̃ do
    c=c̃_α;
    S = ∅;
    g(.) is determined by w;
    for i=1 to k do
        f(.) is determined by c and H_i;
        Initialize solution set X by construct(g(),β,x);
        while termination is not realized do
            Create next generation X_{i+1};
            Copy elite members of X to X_{i+1} (Elitism);
            Select members of X by crossoverprob. × n; pair them up;
            produce offspring; insert the offspring intoX_{i+1} (Crossover);
            Select members of X_{i+1} by mutationprob. × n; invert a
            randomly-selected bit in each (Mutation);
            Repair members of X_{i+1} dont satisfy constraints (Repair);
        end
        Select best x from X by f(.);
        S = S ∪ x;
    end
    S̃ = S̃ ∪ x;
end
```

Algorithm 2: Genetic algorithm with semi-greedy heuristic construction phase

In generation phase of initial population of GA, an individual is iteratively constructed by semi-greedy heuristic which is Greedy Randomized Adaptive Search Procedures (GRASP) construction phase [43]. At each iteration of construction phase, let the set of candidate elements consist of all elements that can be included in the partial solution under construction without destroying feasibility. The selection of the next element for inclusion is determined by the evaluation of all candidate elements according to a greedy evaluation function. This greedy function usually represents the incremental increase in the cost function due to the inclusion of this element into the solution under construction. The evaluation of the elements by this function leads to the creation of a restricted candidate list (RCL) formed by the best but not necessarily top elements (this is the greedy aspect of the algorithm). The element to be included in the partial solution is randomly selected from those in the RCL (this is the probabilistic aspect of the heuristic). Once the selected element is included to the partial solution, the candidate list is updated and the incremental costs are reevaluated (this is the adaptive aspect of the heuristic) [44]. The pseudo code of Algorithm 1 describes construction phase which we use.

The pseudo-code shows that the parameters β controls the balance of between greediness and randomness in the algorithm. β =0 implies a greedy construction procedure while β =1 constructed random solution [45]. In our implementation, β is increased from 0 to 1 in steps of 0,25.

After that, Genetic Algorithm with Semi-Greedy Heuristic Construction Phase (GASGCP) has to be given. The pseudo-code of Algorithm 2 describes algorithm with implementation of Multiple Fuzzy k-cardinality Assignment Problem with HOWA Approach.

5 Numerical Example and Interface

The proposed experimental environment is Windows 7 64-bit version operating system at a 2.3 GHz Core i3 PC machine with 8 GB main memory. First of all, we describe our program which we used to solve numerical example and developed for this study. We used C# programming language along with Microsoft .Net Framework 4.0 under Visual Studio 2010 platform. Our program is user-friendly and simple to use (Fig. 2). This program first developed for multiple k-cardinality assignment problems with HOWA approach without fuzzy parameters. We can utilize the program by adjusting the elements of cost matrix with respect to our problem.

Using the interface, decision-maker may change main components of the problem such as the number

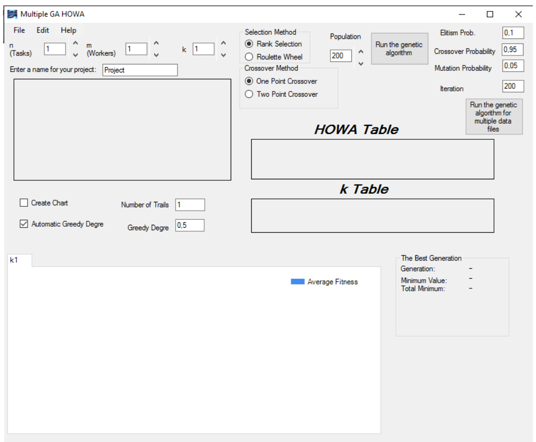


Fig. 2. Graphical user interface of the developed program

of tasks, the number of workers and number of teams. The interface allows decision maker to determine HOWA weights for each team after decision-maker determines main components of the problem. Moreover, it is possible to adjust input characteristics of GA operators such as selection and crossover methods, crossover and mutation probabilities, population size and the number of iterations. Mentioned program make five experiments with different β parameters between 0 and 1, which are performed in parallel, for each parameter set.

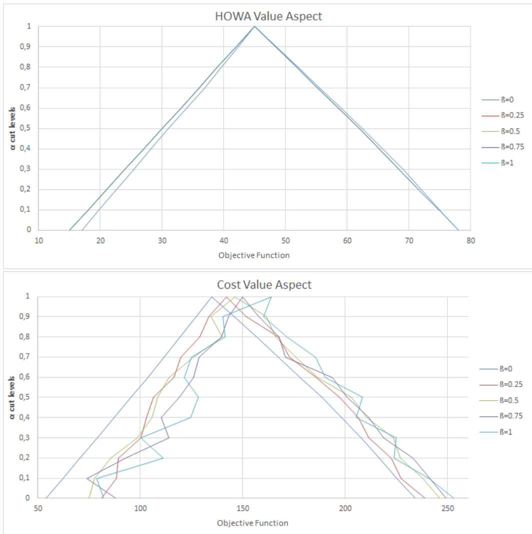


Fig. 3. Graphics of resulting fuzzy numbers for the assignment 3 team with HOWA operators equal min operator

In this section, in order to realize an implementation of the mentioned problem, we employ "gap12" data set, which is used generalized assignment problem, from OR-Library. This data belongs to a crisp problem with 10-workers and 60-tasks. Before implementing the data to our model, we fuzzified it. We fix parameters of GA which is used for solving problem of interest. This fixing process is made by post hoc tests in the Analysis of Variance. Finally, we decided on the following parameters:

- Population size = 200
- Iteration = 200
- Selection method = Rank selection
- Crossover method = One point crossover
- Crossover rate = 0,95
- Mutation rate= 0,05
- Elitism replacement = 20 (10% of the population)

We would like to perform sensitivity analysis concerning the values of HOWA weights and the value of greediness of semi-greedy heuristic (Figs. 3-5). We run our program which is performed five parallel experiments as aforesaid, for each level of mentioned values. After experiments finished, we choose best one of them for result record of belonging values.

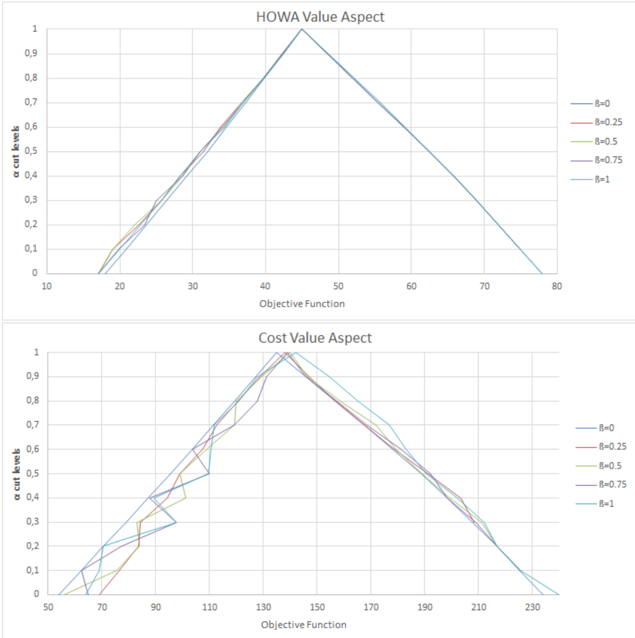


Fig. 4. Graphics of resulting fuzzy numbers for the assignment 3 team with HOWA operators equal median operator

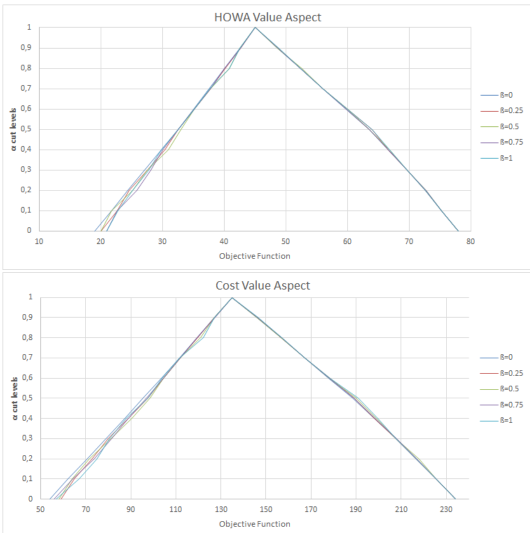
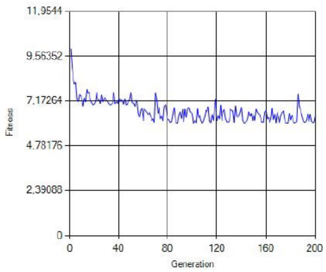
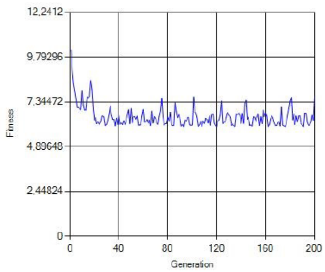


Fig. 5. Graphics of resulting fuzzy numbers for the assignment 3 team with HOWA operators equal max operator

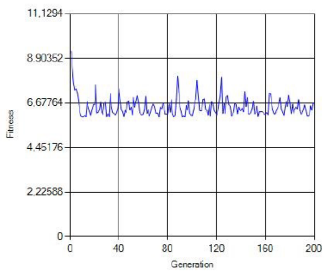
We analyze different particular cases of the HOWA operator such as the min, the max and the median operators. Note that with this analysis, we obtain optimal decisions that depend on the aggregation operator and the value of greediness of semi-greedy heuristic used. Then, we can see that each aggregation operator and the greediness value may lead to different results and decisions. So the decision maker is able to select one or more aggregation operators and different value of greediness of semi-greedy heuristic according to his or her interests.



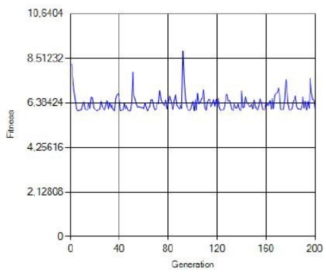
(a) $\beta = 1.0$



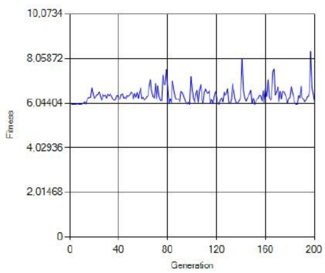
(b) $\beta = 0.75$



(c) $\beta = 0.5$



(d) $\beta = 0.25$



(e) $\beta = 0$

Fig. 6. The plots of the average fitness function of the population versus generations for different β values.

As we can see from Figs. 3 to 5, results must be considered in two different perspectives. By looking at HOWA value aspect, balance between greediness and randomness of construction procedure is shown to be significant. Especially, min and median operators give worst results when level of greediness or randomness is high. In the max operator, this impact is observed hardly.

Table 1. Comparison between Cost value and HOWA value aspects based on β values

β	Cost Value	HOWA Value
1	127,6	18
0,75	90,7	18
0,5	88,3	18
0,25	73,9	19,8
0	62,3	19,8

We reach slightly different results when cost value which is calculated by some solution at HOWA, aspect is taken a matter in hand. When dealing with min and median operators, high levels of greediness creates better and more regular shaped fuzzy numbers. But the max operator gives some results regardless of greediness level.

As seen in Fig. 6, different β values which cause levels of greediness, have an impact on the evolution of the algorithm. Here, if β value converges to 1.0, the diversity of initial population increases. On the other hand, when β value converges to 0.0, the diversity of initial population decreases. According to the complexity of the related problems, determining β value affects convergence speed and quality of optimal solution. The balance between convergence speed and quality have to be maintained by choosing correct β value.

Table 1 shows that β values impress the relationship between cost and HOWA value aspect. If decision maker considers cost values as well as HOWA values, he/she can take into account that β values impact balance between cost and HOWA values. Considering trade-off between cost values and HOWA values, the critical value of β has to be somewhere between 0.5 and 0.25 for our example.

According to aforementioned observations, concerning quality of the optimal solution, we can say that quality of the optimal solution could be related two aspects:

(i) β
(ii) Complexity of the interested problem

While the related problem is complex and if decision maker would like to reach optimal solution as fast as possible, it is recommended decision maker to pick β value close (but not equal) to 0.0. But decision maker would sacrifice the quality of optimal solution a little bit in that setup.

6 Conclusion

The assignment problem, which is a common problem in the real world a, is one of the well-known combinatorial optimization problems. In recent years, assignment problem with side constraint attracts much attention among scientists. In general, this problem may be evaluated as teaming up a group among bunch of workers. In this study, we mention more general case as building more than one team. Moreover, we state the model that allows decision-maker to measure performance of each team with different scale by employing HOWA operator. We also use fuzzy theory to model uncertainty originated from the elements of cost matrix.

We developed a program with an interface to apply this model to data set. We employed “gap12” data set from OR-Library. We run the program for different weights and greediness value of the problem. We observe that changing HOWA weights has great influence on resulting fuzzy number.

This is what we expected from the model and this property allows decision-maker to choose between alternative solutions with respect to his/her own decision making strategy. On the other hand, influence on the results is changed or fixed in line with our goal by semi-greedy construction phase. But HOWA operator begin to resemble max operator, it acquires resistance against semi-greedy construction phase. So we can affect results in any tape of HOWA operators without close to the max operator by changing greediness or randomness level of construction initial population of GA. Also, construction phase of initial population influences convergence speed of the algorithm. However, it should be evaluated that there is a trade-off between convergence speed and quality of solution.In addition to convergence speed of the algorithm, The construction phase allows decision makers to make more flexible decisions and desired trade-off about different aspects.

Competing Interests

Authors have declared that no competing interests exist.

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