

# Finite Element Analysis of Magnetohydrodynamic Natural Convection within Semi-Circular Top Enclosure with Triangular Obstacles

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## Abstract

The phenomena of magneto-hydrodynamic natural convection in a two-dimensional semicircular top enclosure with triangular obstacle in the rectangular cavity were studied numerically. The governing differential equations are solved by using the most important method which is finite element method (weighted-residual method). The top wall is placed at cold  $T_c$  and bottom wall is heated  $T_h$ . Here the sidewalls of the cavity assumed adiabatic. Also all the wall are occupied to be no-slip condition. A heated triangular obstacle is located at the center of the cavity. The study accomplished for Prandtl number  $Pr = 0.71$ ; the Rayleigh number  $Ra = 10^3, 10^5, 5 \times 10^5, 10^6$  and for Hartmann number  $Ha = 0, 20, 50, 100$ . The results represent the streamlines, isotherms, velocity and temperature fields as well as local Nusselt number.

## Keywords

Natural Convection, Finite Element Method, Hartmann Number, Rayleigh Number, Triangular Obstacle, Numerical Solution

## 1. Introduction

Convection is a mode of heat transfer which takes place through the movement of collective masses of heated atoms and molecules within fluids such as gases and liquids, including molten rock. Application of natural convection heat transfer is very important in science, engineering researcher and fields such as thermal insulation, heating and cooling buildings, solar collector, heat exchang-

er, crystal growing, food processing, energy drying processes, lakes and geothermal reservoirs, nuclear energy, underground water flow, etc. Several numerical and experimental systems have been advanced to investigate flow characteristics inside the cavities with and without obstacles. Most of the cavities repeatedly used in industries are rectangular, square, trapezoidal, cylindrical, elliptic and triangular, etc.

Earlier studies were mainly developed on physics of the various flow systems in different cavity. Reddy [1] introduced finite element analysis to develop the energy and momentum equations subject to the boundary conditions simultaneously and the finite element solutions of differential equations with constant coefficients are exact at the nodes. Hussein *et al.* [2] developed entropy generation analysis of a natural convection inside a sinusoidal surrounding with various shapes of cylinders. Their outcomes showed that the entropy generations due to heat transfer, fluid friction and total entropy generation enhance with increasing values of Rayleigh number, while the local Bejan number decreases. Arun *et al.* [3] investigate on natural convection heat transfer problems by Lattice Boltzmann Method. They found the importance of the natural convection problem and the ability of lattice Boltzmann method to apply in the computational field. Seyyedi *et al.* [4] studied natural convection heat transfer under constant heat flux wall in a nanofluid filled annulus enclosure. Sheikholeslami *et al.* [5] investigated natural convection heat transfer in a nanofluid filled inclined L-shaped enclosure. It can be terminated that the turning angle of the enclosure can be a control parameter for heat and fluid flow. The outcomes publish that average Nusselt number is an increasing function of nanoparticle volume fraction and Rayleigh number. Natural convection in a triangular top wall enclosure with a solid strip was extensively experimentally and numerically researched by Hussain *et al.* [6]. Bhuiyan *et al.* [7] studied magneto hydrodynamic natural convection in a square cavity with semicircular heated obstacle. They investigated the effect of Rayleigh and Hartmann numbers on the flow field in a square cavity with semicircular heated block along uniform magnetic field. Numerical simulation of natural convection in a rectangular lacuna with triangles of different orientation in presence of magnetic field was researched by Alam *et al.* [8]. They also establish the mentioned parameters that have meaningful effect on average Nusselt number at the hot wall and average temperature of the fluid in the enclosure. Saika *et al.* [9] investigated effect of Hartmann Number on Free Convective Flow of MHD Fluid in a Square Cavity with a Heated Cone of Different Orientation. They reported that a sufficiently large magnetic field can potentially stop fluid movement altogether and heat transfer would be fully by conduction. Hakan and Khaled *et al.* [10] studied MHD mixed convection in a lid-driven cavity with corner heater. Finite Element Analysis of Magnetohydrodynamic Mixed Convection in a Lid-Driven Trapezoidal Enclosure Having Heated Triangular Block was researched by Sajjad and Alim *et al.* [11].

The above literature review motivates the author's, to give attention to the

problem of natural convection in enclosure with obstacles of different shapes like triangles, circles, solid strips and so on. To the best of author's knowledge no investigation has been done yet on finite element analysis of MHD natural convection within semi-circular top enclosure with triangular obstacle. So the proposed study is to address the issue.

## 2. Physical Configuration

The physical model deliberated in the present study of a two-dimensional rectangular cavity and semi-circular top enclosure with heated triangular obstacles is shown in FIGURE1 is considered for simulation purposes. The height and the width of the cavity are denoted by  $L$ . The lower wall is kept at heated ( $T_h$ ) and the upper wall is kept at cold ( $T_c$ ) under all situations  $T_h > T_c$  condition is maintained. The right and left wall are adiabatic. The gravitational force  $g$ , acts vertically downward. The magnetic field of strength  $B_0$  is applied parallel to the  $x$ -axis.

## 3. Mathematical Formulation

The flow is considered steady, laminar, incompressible and two-dimensional. The field of governing equations solved during the simulation for the free convection flow inside the domain are, conservation of mass, momentum and energy can be written as:

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum Equations

$$\rho \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right) = -\frac{\partial u}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (2)$$

$$\rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = -\frac{\partial v}{\partial y} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \rho g \beta (T - T_c) - \sigma B_0^2 v \quad (3)$$

Energy Equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (4)$$

where  $x$  and  $y$  are the distances measured along the horizontal and vertical directions respectively;  $u$  and  $v$  are the velocity components in the  $x$  and  $y$  directions respectively;  $T$  denote the fluid temperature,  $T_c$  denotes the reference temperature for which buoyant force vanishes,  $\rho$  is the fluid density,  $g$  is the acceleration due to gravity,  $\beta$  is the volumetric coefficient of thermal expansion,  $\sigma$  is the electrical conductivity,  $B_0$  is the magnetic induction,  $\alpha$  is the thermal diffusivity and  $\nu$  kinematic viscosity of the fluid.

The governing equations are non-dimensionalized using the following dimensionless variables:

$$X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{uL}{\alpha}, V = \frac{vL}{\alpha}$$

$$P = \frac{\rho L^2}{\rho \alpha^2}, \theta = \frac{T - T_c}{T_h - T_c}, \sigma = \frac{\rho^2 \alpha}{L^2}, \alpha = \frac{k}{\rho C_p}$$

Introducing the previous dimensionless variables, the following dimensionless forms of the governing equation are obtained as follow:

Continuity Equation

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (5)$$

Momentum Equations

$$U \frac{\partial U}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial X} + Pr \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (6)$$

$$U \left( \frac{\partial V}{\partial X} + \frac{\partial V}{\partial Y} \right) = -\frac{\partial P}{\partial Y} + Pr \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{Ra}{Pr} \theta - Ha^2 Pr V \quad (7)$$

Energy Equation

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (8)$$

where  $X$  and  $Y$  are the coordinates varying along horizontal and vertical directions, respectively,  $U$  and  $V$  are the velocity components in the  $X$  and  $Y$  directions, respectively,  $\theta$  is the dimensionless temperature and  $P$  is the dimensionless pressure.  $C_p$  is the fluid specific heat at constant pressure;  $k$  is the thermal conductivity of fluid.

The dimensionless parameters are the Prandtl number  $Pr$ , Hartmann number  $Ha$  and Rayleigh number  $Ra$ . which are defined as:

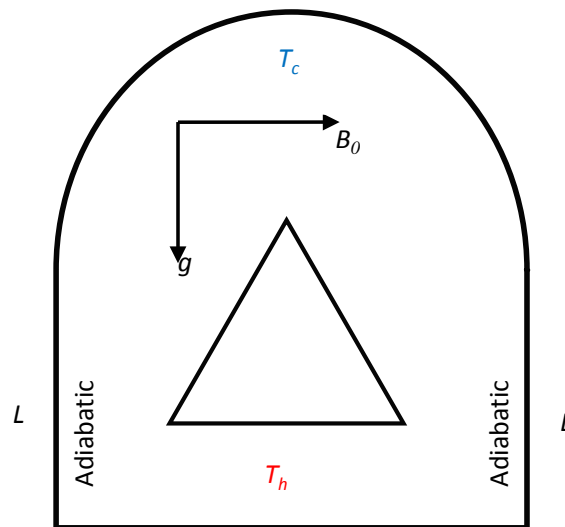
$$Pr = \frac{\nu}{\alpha}, Ha^2 = \frac{\sigma B_0^2 L^2}{\mu}, Ra = \frac{g \beta L^3 (T_h - T_c) Pr}{\nu^2}$$

### Boundary Conditions

The simulation domain with boundary conditions is shown in **Figure 1**. Which are No-slip conditions are applied at all cavity boundaries, *i.e.*  $u = v = 0$ . The left and right sidewalls are considered adiabatic. The lower wall is subjected to isothermal heat temperature ( $T_h$ ). The upper wall is subjected to isothermal cold temperature ( $T_c$ ). The internal triangular obstacle is taken thermally insulated everywhere, *i.e.*  $\frac{dT}{dy} = 0$ .

### 4. Numerical Procedure

The dimensionless governing equations are solved with the required boundary conditions using the finite element method, the Galerkin weighted residual technique. Mass, momentum and energy are two dimensional non-linear partial



**Figure 1.** Schematic physical configuration with boundary condition considered in the present paper.

differential equations which are transferred into a system of integral equations. The quadratic triangular element is used to develop the finite element equations. Different types of grid densities have been selected to assess the accuracy of the numerical simulation procedure. These modified nonlinear equations are transferred in linear algebraic equations with the aid of Newton's method. Finally, the triangular factorization method is applied for solving those linear equations.

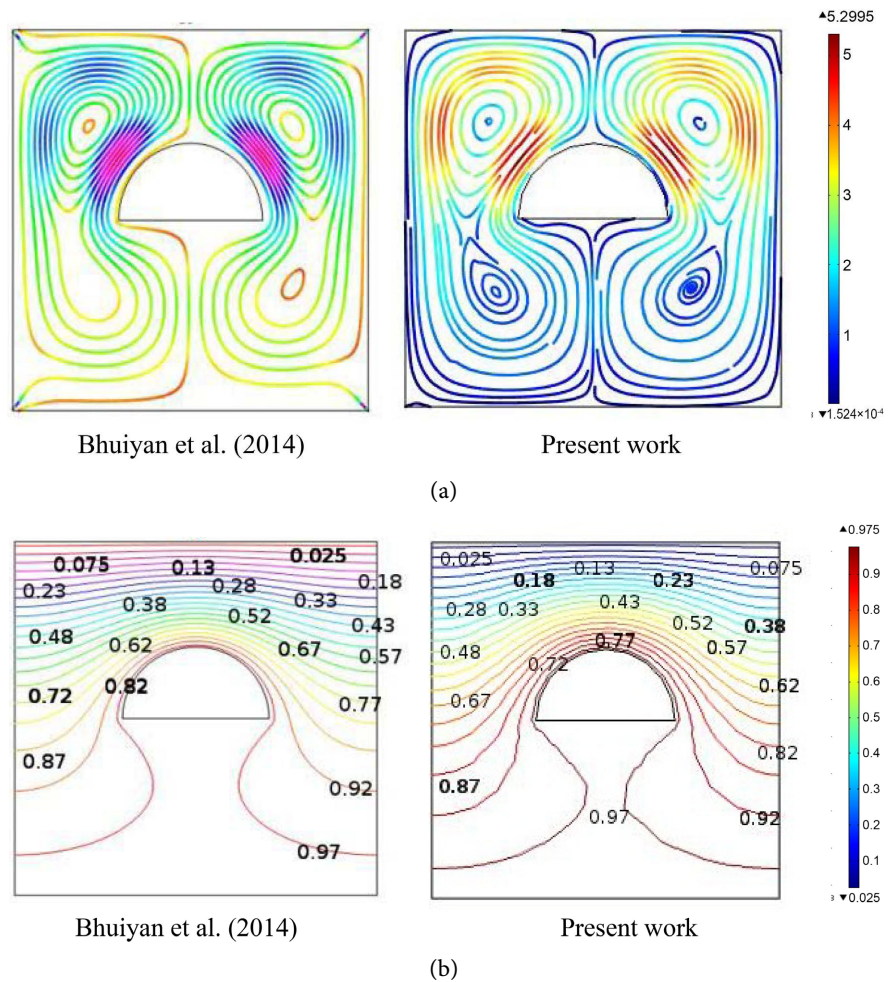
## 5. Validation of Numerical Procedure

Validation of the numerical procedure was made by comparing streamlines and isotherms with results shown in **Figure 2(a)** and **Figure 2(b)** by Bhuiyan *et al.* (2014). They investigate the effect of magnetic field in a square cavity with semi-circular heated block. From this figures as seen the obtained outcomes show excellent agreement.

## 6. Results and Discussion

Following portion the estimates are completed for Prandtl number  $Pr = 0.71$ , the Rayleigh number  $Ra = 10^3, 10^5, 5 \times 10^5, 10^6$  and for Hartmann number  $Ha = 0, 20, 50, 100$ . The outcomes are pictorial with isotherms, streamlines, velocity profiles, dimensionless temperature and the local Nusselt number.

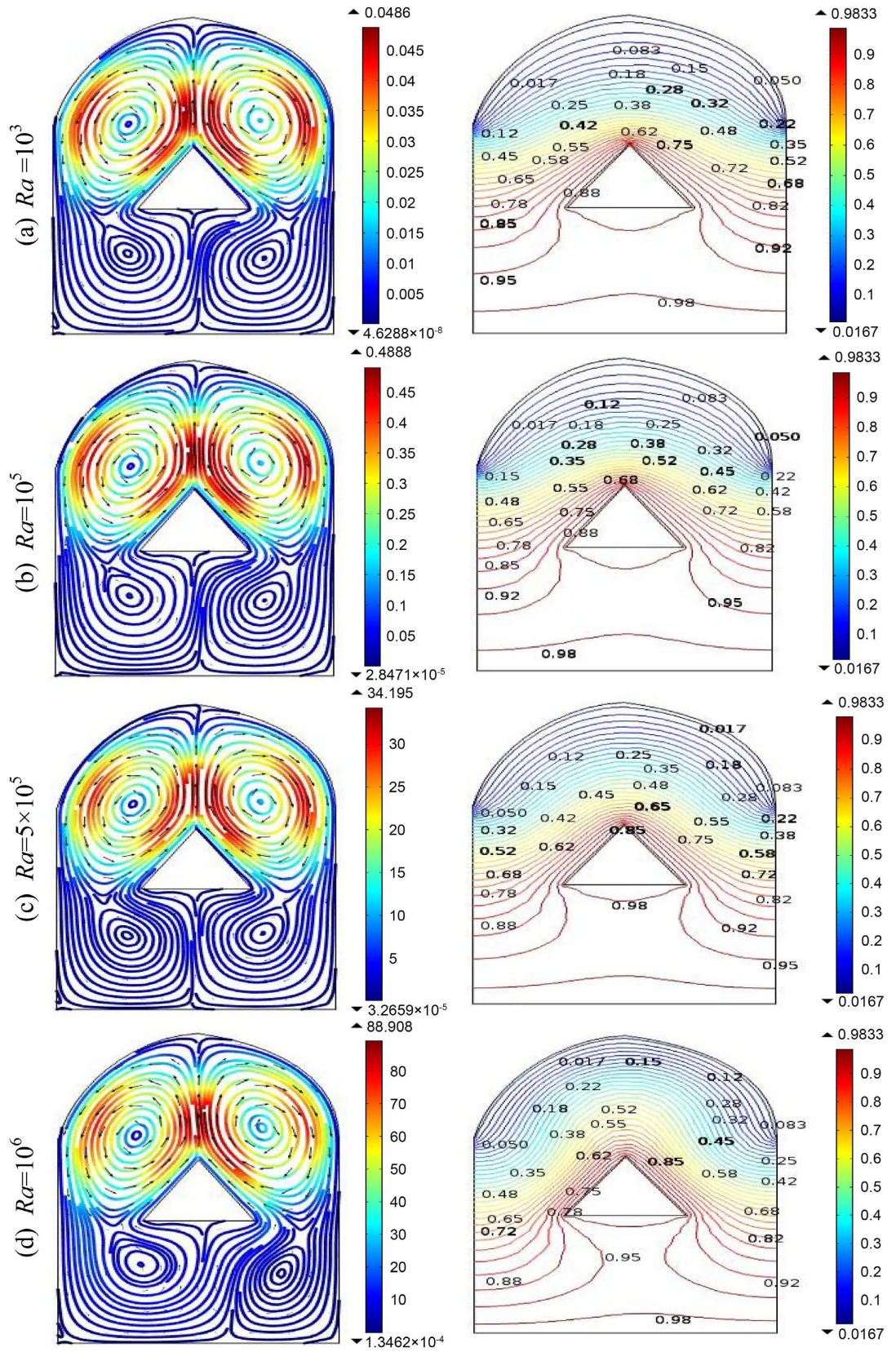
Streamlines and Isotherms are shown in **Figure 3**. From the streamlines figure, many cells can be observed inside the cavity. For  $Ha = 0$  where there is no magnetic field are presented in **Figure 3** to comprehend the effects of Rayleigh number on the flow field and temperature distribution. At  $Ra = 10^3$  and in the non-existence of the magnetic field ( $Ha = 0$ ) are created four elliptic-shaped cells appear on the top-right, top-left, bottom-right and bottom-left corner of the triangular heated block of the cavity shown in the left side of **Figure 3(a)**. The top-left cells rotate anticlockwise and the top-right cells rotate clockwise. For



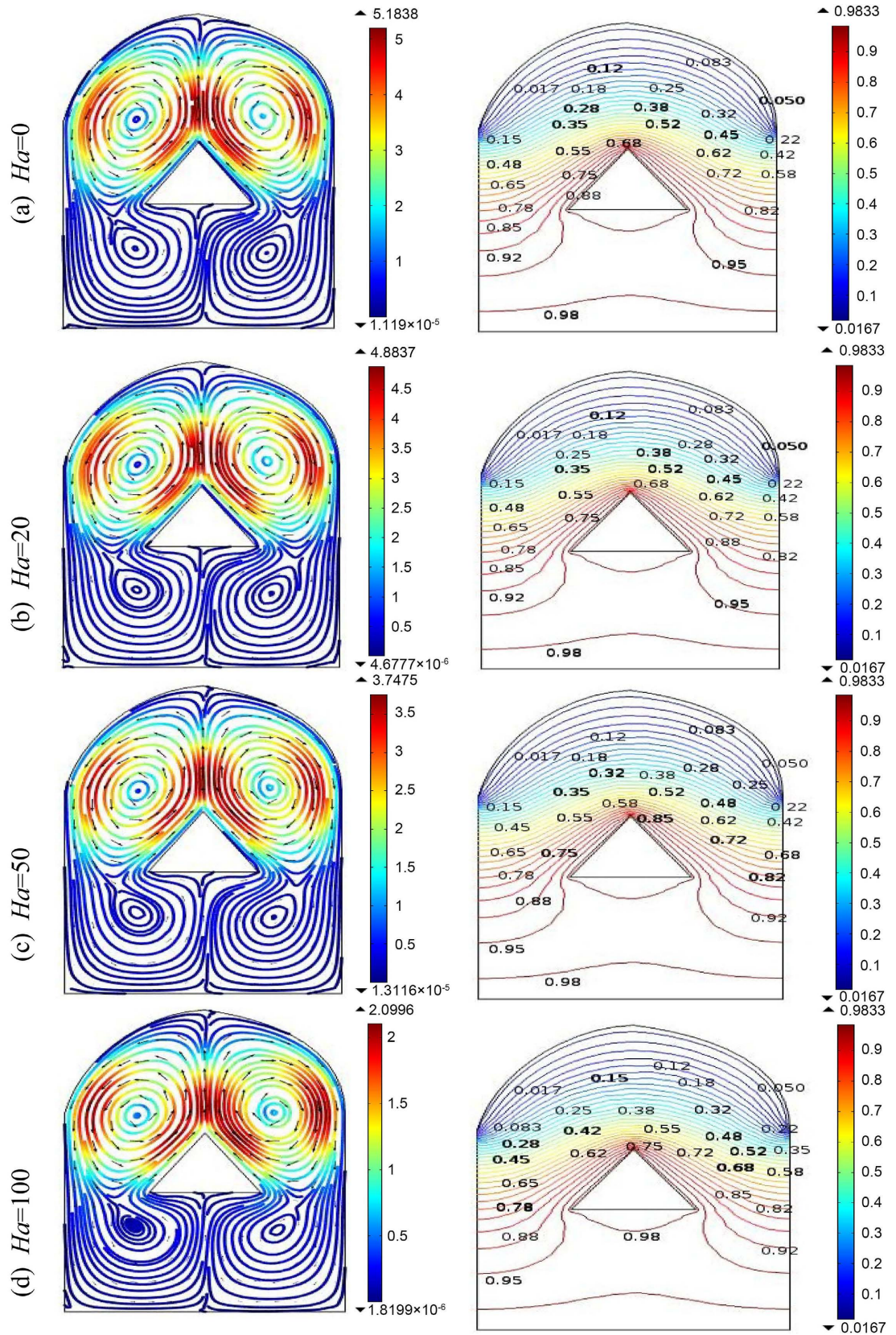
**Figure 2.** (a) Comparison results for Streamlines while  $Ha = 0$ ,  $Pr = 0.71$  and  $Ra = 10^4$ ; (b) Comparison results for Isotherms while  $Ha = 0$ ,  $Pr = 0.71$  and  $Ra = 10^4$ .

higher Rayleigh numbers almost the similar outcomes as **Figure 3(a)** but flow power increases are shown in **Figures 3(b)-(d)**. The right side of **Figure 3** shows the isotherms for different values of Rayleigh number ( $Ra$ ) with  $Pr = 0.71$  and  $Ha = 0$ . The dimensionless temperature whose value range is 0 - 1. Stream function has a symmetrical value about the vertical center line as the triangular heated obstacle is symmetrical. It is observed that isothermal lines slightly move from the heated surfaces to cold surfaces on the right side of **Figure 3(a)** and **Figure 3(b)** at  $Ra = 10^3$  and  $Ra = 10^5$ . With increasing Rayleigh number, isotherms are many submission which means increasing heat transfer through convection. The differences are clean between  $Ra = 5 \times 10^5$  and  $Ra = 10^6$  are shown on the right side of **Figure 3(c)** and **Figure 3(d)**.

Streamlines and Isotherms are shown in **Figure 4** where Rayleigh number ( $Ra = 10^5$ ) is fixed.  $Ha$  is increased to 0, 20, 50 and 100. At  $Ra = 10^5$  and increasing  $Ha = 0$  to 100 are created four elliptic-shaped cells be present on the top-right, top-left, bottom-right and bottom-left corner of the triangular heated block of the cavity shown in the left side of **Figure 4(a)**. The top-left cells rotate anticlockwise



**Figure 3.** Streamlines and isotherms for (a)  $Ra = 10^3$ ; (b)  $Ra = 10^5$ ; (c)  $Ra = 5 \times 10^5$ ; (d)  $Ra = 10^6$  while  $Ha = 0$  &  $Pr = 0.71$ .

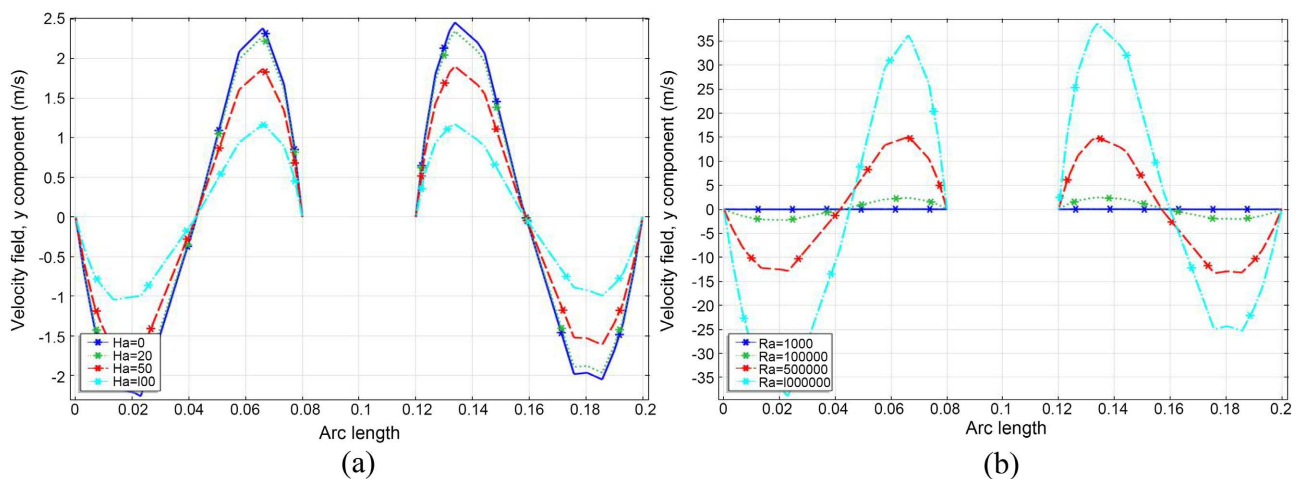


**Figure 4.** Streamlines and isotherms for (a)  $Ha = 0$ ; (b)  $Ha = 20$ ; (c)  $Ha = 50$ ; (d)  $n = 100$  while  $Ra = 10^5$  &  $Pr = 0.71$ .

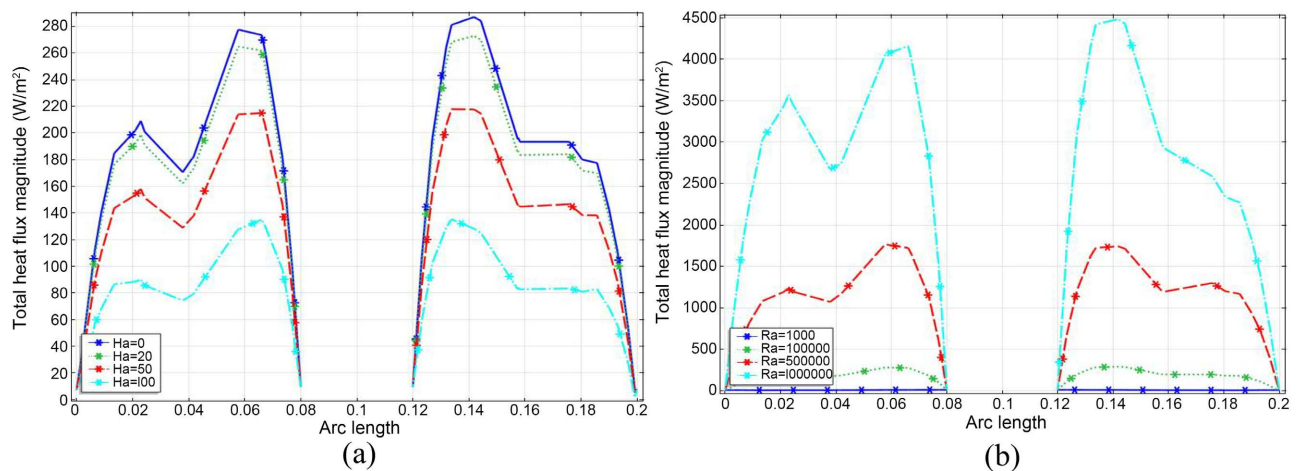


and the top-right cells rotate clockwise. For higher Hartmann numbers almost the same result as **Figures 4(a)-(c)** but flow power increases and closer to one another are shown in **Figure 4(d)**. The right side of **Figure 4** shows the isotherms for various values of Hartmann number ( $Ha$ ) with  $Pr = 0.71$  and  $Ra = 10^5$ . Stream function has a symmetrical value about the vertical centerline as the triangular heated obstacle is symmetrical. It is observed that  $Ha$  does not have more effect on heat transfer with high  $Ra$ . The differences are clean between  $Ha = 50$  and  $Ha = 100$  are shown on the right side of **Figure 4(c)** and **Figure 4(d)**.

The  $y$ -component of velocity along the line parallel to the  $x$  axis ( $y = 0.15$ ) is presented. **Figure 5(a)** and **Figure 5(b)** display the effect of different Rayleigh number ( $Ra$ ) and Hartmann number ( $Ha$ ) with  $Pr = 0.71$  on the  $y$ -component of velocity at  $y = 0.15$ . **Figure 5(a)** display the four profiles corresponding to the four values of  $Ha$  which are 0, 20, 50 and 100. It can be visible from **Figure 5(a)** that the velocity has a larger change for the lower Hartmann number value. Velocity



**Figure 5.** Variation of velocity profiles at different (a) Hartmann number with  $Pr = 0.71$ ,  $Ra = 10^5$  and (b) Rayleigh number with  $Pr = 0.71$  and  $Ha = 0$  on  $y$ -component at  $y = 0.15$ .



**Figure 6.** Variation of local nusselt number at different (a) Hartmann number with  $Pr = 0.71$ ,  $Ra = 10^5$  and (b) Rayleigh number with  $Pr = 0.71$  and  $Ha = 0$  on  $y$ -component at  $y = 0.15$ .

grows maximum at two locations which is at  $x = 0.06$  and  $x = 0.14$  when  $Ha = 0$ . **Figure 5(b)** displays the four profiles corresponding to the four values of Rayleigh number ( $Ra$ ) which are  $10^3$ ,  $10^5$ ,  $5 \times 10^5$  and  $10^6$ . It can be visible from **Figure 5(b)** that velocity has a big change for a higher Rayleigh number value. As seen from **Figure 5(a)** and **Figure 5(b)** there is no value of velocity on the triangular heated block. The local Nusselt number at different Hartmann number ( $Ha$ ) and Rayleigh number ( $Ra$ ) with  $Pr = 0.71$  on  $y$ -component at  $y = 0.15$  is shown in **Figure 6(a)** and **Figure 6(b)**. At  $Ha$  minimum, we get maximum values and at  $Ha$  maximum, we get minimum shape curve which is shown in **Figure 6(a)**. For different Hartmann numbers, we get different curves. Again the local Nusselt number at a various Rayleigh number with  $Pr = 0.71$  and  $Ha = 0$  fixed on the  $y$ -component at  $y = 0.15$  is shown in **Figure 6(b)**. At  $Ra$  maximum we get maximum shape curve and if  $Ra$  minimum we get the minimum value.

## 7. Conclusion

A numerical study investigates Magneto-hydrodynamic natural convection fluid flow and heat transfer in a semi-circular top square cavity filled with heated triangular obstacles. The fluid considered is air. The governing equation of mass, energy equation and momentum were solved using the Galerkin weighted residual method of finite element method. For all cases considered four counter-rotating eddies were shaped inside the cavity. From the present investigation the following conclusions may be drawn as: Rate of heat transfer enhanced for the increase in the buoyancy force via an increase in Rayleigh number with  $Ha = 0$  and reduced for the increasing value of Hartmann number. And also magnetic field strength plays a significant role to control parameters for heat transfer and fluid flow.

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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