



## Kamal Adomian Decomposition Method for Solving Nonlinear Wave-Like Equation with Variable Coefficients

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*Author's contribution*

*The sole author designed, analyzed and interpreted and prepared the manuscript.*

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### Abstract

In this paper, we applied a new method for solving nonlinear wave-like equation with variable coefficients, when the exact solution has a closed form. This method is Kamal Adomian Decomposition Method (KADM). The Kamal decomposition method is a combined form of the Kamal transform method and the Adomian decomposition method [1,2,3]. The nonlinear term can easily be handled with the help of Adomian polynomials which is considered to be a significant advantage of this technique over the other methods. The results reveal that the Kamal decomposition method is very efficient, simple and can be applied to other nonlinear problems.

*Keywords: Kamal transform; Kamal Adomian decomposition; Wave-like equation.*

### 1 Introduction

Kamal transform is an integral transform similar to the Sumudu transform and Elzaki transform. Kamal integral transform was introduced in 2016 by Kamal to solve differential equations and some time domain problems in engineering as in [4], Kamal Transform is derived from the Fourier integral in simple and logical mathematical steps [4]. The purpose of this paper is to show the applicability and efficiency of this interesting new integral transform in solving wave-like differential equations with variable coefficients.

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Despite of many effort given to find closed form solution for non-linear partial differential equation , still it is not easy to obtain closed-form solutions for most of real-life problems that modeled as non-linear partial differential equation. A wide class of analytical methods and numerical methods were introduced to treat such problems. In recent years, various methods have been used such as finite difference method [5], Adomian decomposition method [1,2], the iteration method [6,7], integral transform [8,9], weighted finite difference techniques [5], Laplace decomposition method [10] and homotopy perturbation method [11,5, 8].

On the light of the ongoing research in this area, we describe the application of Kamal decomposition method for nonlinear wave-like equation with initial condition based on the analysis by previous researchers.

From the previous relative studies one can find a broad class of methods dealing with the problem of approximate solutions to problems described by nonlinear fractional differential equations. The perturbation methods have some limitations; for instance, the approximate solution engages series of small parameters which causes difficulty since most nonlinear problems have no small parameters at all. Even though a suitable choice of small parameters occasionally lead to ideal solution, in most cases unsuitable choices leads to serious effects in the solutions . Therefore, an analytical method which does not require a small parameter in the equation modeling of the problem is preferred [11,7]. To deal with the deficiency accompanied these perturbation methods for solving nonlinear equations. The homotopy perturbation method (HPM) was first initiated in [12,13]. The HPM was also studied by many authors to present approximate solution of linear and nonlinear equations arising in various fields [6,13,14]. The Adomian decomposition method [1,2] , and variational iteration method (VIM) [7] have also been applied to study the various physical problems. The , (HDM) was recently proposed by [1,2] to solve the groundwater flow equation [11]. The homotopy decomposition method is actually the combination of the perturbation method and Adomian decomposition method. Singh et al. [13] have made used of studying the solutions of linear and nonlinear partial differential equations by using the homotopy perturbation transform method (HPSTM) [15,12,13] . The HPSTM is a combination of Sumudu transform, HPM, and He’s polynomials.

The Kamal Transform of partial derivative is derived, and its applicability was demonstrated using four different partial equations in [16] . Based on the work in [16] we apply KADM to solve nonlinear wave-like equations with variable coefficients.

## 2 Definition and Derivations of Kamal Transform of Derivatives

Kamal transform of the function  $f(t)$  is defined as

$$[f(t)] = \int_0^\infty f(t)e^{-\frac{t}{v}} dt = G(v) , t > 0 , k_1 \leq v \leq k_2 \tag{1}$$

To obtain Kamal transform of partial derivatives we use integration by parts as follows :

$$\begin{aligned} K \left[ \frac{\partial f(x,t)}{\partial t} \right] &= \int_0^\infty \frac{\partial f(x,t)}{\partial t} e^{-\frac{t}{v}} dt = \lim_{p \rightarrow \infty} \int_0^p e^{-\frac{t}{v}} \frac{\partial f(x,t)}{\partial t} dt \\ &= \lim_{p \rightarrow \infty} \left( \left[ e^{-\frac{t}{v}} f(x,t) \right]_0^p + \frac{1}{v} \int_0^p e^{-\frac{t}{v}} f(x,t) dt \right) \\ &= -f(x,0) + \frac{1}{v} G(x,v) \end{aligned}$$

Thus,

$$K \left[ \frac{\partial f(x,t)}{\partial t} \right] = v^{-1} G(x,v) - f(x,0) \tag{2}$$

To find

$$K \left[ \frac{\partial^2 f(x,t)}{\partial t^2} \right], \text{ let } \frac{\partial f(x,t)}{\partial t} = g(x,t) , \text{ then by using Eq. (2) we have:}$$

$$K \left[ \frac{\partial^2 f(x,t)}{\partial t^2} \right] = K \left[ \frac{\partial g(x,t)}{\partial t} \right] = v^{-1} K \left[ \frac{\partial g(x,t)}{\partial t} \right] - g(x,0)$$

$$K \left[ \frac{\partial^2 f(x,t)}{\partial t^2} \right] = v^{-2} G(x,v) - v^{-1} f(x,0) - \frac{\partial f(x,0)}{\partial t} \tag{3}$$

we can easily extend this result to the *n*th partial derivative by using mathematical induction.

Now, we assume that  $f(x, t)$  is piecewise continuous and is of exponential order.

Then

$$K \left[ \frac{\partial f(x,t)}{\partial x} \right] = \int_0^\infty e^{-\frac{t}{v}} \frac{\partial f(x,t)}{\partial x} dt$$

Using the Leibniz' rule

$$K \left[ \frac{\partial f(x,t)}{\partial x} \right] = \int_0^\infty e^{-\frac{t}{v}} \frac{\partial f(x,t)}{\partial x} dt = \frac{\partial}{\partial x} \int_0^\infty e^{-\frac{t}{v}} f(x,t) dt$$

Thus ,

$$K \left[ \frac{\partial f(x,t)}{\partial x} \right] = \frac{d}{dx} (G(x, v)) \tag{4}$$

Also we can find:

$$K \left[ \frac{\partial^2 f(x,t)}{\partial x^2} \right] = \frac{d^2}{dx^2} (G(x, v)) \tag{5}$$

And

$$K \left[ \frac{\partial^n f(x,t)}{\partial x^n} \right] = \frac{d^n}{dx^n} (G(x, v)) \tag{6}$$

### 3 Basic Idea of Kamal Adomian Decomposition Method (KADM)

The general form of nonlinear non-homogeneous partial differential equation can be considered as the follow:

$$Du(x, t) + Ru(x, t) + Nu(x, t) = f(x, t) \tag{7}$$

With the following initial conditions

$$u(x, 0) = h(x), \quad u_t(x, 0) = g(x)$$

Where D is the second order linear differential operator  $= \frac{\partial^2}{\partial t^2}$ , is the linear differential operator of less order than D, N represents the general non-linear differential operator and  $f(x,t)$  is the source term.

Taking Kamal transform (denoted throughout this paper by  $K(\cdot)$ ) on both sides of Eq. (7), to get:

$$K[Du(x, t)] + K[Ru(x, t)] + K[Nu(x, t)] = K[f(x, t)] \tag{8}$$

Using the differentiation property of Kamal transform and above initial conditions, we have:

$$K[u(x, t)] = v^2 K[f(x, t)] + vh(x) + v^2 g(x) - v^2 K[Ru(x, t) + Nu(x, t)] \tag{9}$$

Operating with the Kamal inverse transform as in [16] , on both sides of Eq.(9) gives:

$$u(x, t) = G(x, t) - K^{-1}[v^2K[Ru(x, t) + Nu(x, t)]] \tag{10}$$

Where  $G(x, t)$  represents the term arising from the source term and the prescribed initial condition.

Now, we apply the Adomian decomposition method as in [10]

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t). \tag{11}$$

And the nonlinear term can be decomposed as:

$$Nu(x, t) = \sum_{n=0}^{\infty} A_n(u) \tag{12}$$

Where  $A_n(u)$  are Adomian polynomials of  $u_0, u_1, u_2 \dots u_n$  and it given by the following formula:

$$A_n = \frac{1}{n!} \frac{\partial^n}{\partial \lambda^n} [N(\sum_{i=0}^{\infty} \lambda^i u_i)]_{\lambda=0}, n = 0, 1, 2, \dots \tag{13}$$

Substituting Eqs. (12) and (11) in Eq. (10) we get:

$$\sum_{n=0}^{\infty} u_n(x, t) = G(x, t) - K^{-1}[v^2K[R \sum_{n=0}^{\infty} u_n(x, t) + \sum_{n=0}^{\infty} A_n(u)]] \tag{14}$$

On comparing both sides of the Eq. (14), we get

$$\begin{aligned} u_0(x, t) &= G(x, t), \\ u_1(x, t) &= -K^{-1}[v^2K[Ru_0(x, t) + A_0(u)]], \\ u_2(x, t) &= -K^{-1}[v^2K[Ru_1(x, t) + A_1(u)]], \\ u_3(x, t) &= -K^{-1}[v^2K[Ru_2(x, t) + A_2(u)]], \\ &\vdots \end{aligned}$$

In general the recursive relation is given by

$$\begin{aligned} u_0(x, t) &= G(x, t), \\ u_{n+1}(x, t) &= -K^{-1}[v^2K[Ru_n(x, t) + A_n(u)]] , n \geq 0 \end{aligned} \tag{15}$$

Finally, applying Kamal transform of the right hand side Eq.(15) and then taking inverse Kamal transform, we get  $u_0, u_1, u_2, \dots$  which are the series form of the desired solutions.

## 4 Numerical Examples

In this section we discuss some examples to illustrate Kamal transform Decomposition Method.

### Example 4.1:

Let's consider the second-dimensional nonlinear wave-like equation with variable coefficients [14].

$$u_{tt} = \frac{\partial^2}{\partial x \partial y} (u_{xx} u_{yy}) - \frac{\partial^2}{\partial x \partial y} (xy u_x u_y) - u \tag{16}$$

With the initial condition;

$$u(x, y, 0) = e^{xy}, u_t(x, y, 0) = e^{xy} \tag{17}$$

Applying Kamal transform of both sides of Eq. (16) , we obtain,

$$K[u_{tt}(x, y, t)] = K \left[ \frac{\partial^2}{\partial x \partial y} (u_{xx}u_{yy}) - \frac{\partial^2}{\partial x \partial y} (xyu_xu_y) - u \right] \quad (18)$$

Using the differential property of Kamal transform . Eq. (18) can be written as:

$$K[u(x, y, t)] = ve^{xy} + v^2e^{xy} + v^2K \left[ \frac{\partial^2}{\partial x \partial y} (u_{xx}u_{yy}) - \frac{\partial^2}{\partial x \partial y} (xyu_xu_y) - u \right] \quad (19)$$

The inverse of Kamal transform implies that:

$$u(x, y, t) = e^{xy} + te^{xy} + K^{-1} \left[ v^2K \left[ \frac{\partial^2}{\partial x \partial y} (u_{xx}u_{yy}) - \frac{\partial^2}{\partial x \partial y} (xyu_xu_y) - u \right] \right] \quad (20)$$

Following the technique, if we assume an infinite series solution of the form Eqs.(11) and (12), we obtain

$$\sum_{n=0}^{\infty} u_n(x, y, t) = e^{xy} + te^{xy} + K^{-1} \left[ v^2K \left[ \frac{\partial^2}{\partial x \partial y} \sum_{n=0}^{\infty} A_n(u) - \frac{\partial^2}{\partial x \partial y} (xy \sum_{n=0}^{\infty} B_n(u)) - \sum_{n=0}^{\infty} u_n(x, y, t) \right] \right] \quad (21)$$

Where  $A_n(u)$  and  $B_n(u)$  are Adomian polynomials that represent nonlinear term. So Adomian polynomials are given as follows:

$$A_n(u) = u_{xx}u_{yy} \text{ and } B_n(u) = u_xu_y$$

Using (13). The few components of the Adomian polynomials are given as follows:

$$\begin{aligned} A_0 &= (u_0)_{xx}(u_0)_{yy} \\ A_1 &= (u_1)_{xx}(u_0)_{yy} + (u_0)_{xx}(u_1)_{yy} \\ A_2 &= (u_2)_{xx}(u_0)_{yy} + (u_1)_{xx}(u_1)_{yy} + (u_0)_{xx}(u_2)_{yy} \\ B_0 &= (u_0)_x(u_0)_y \\ B_1 &= (u_1)_x(u_0)_y + (u_0)_x(u_1)_y \\ B_2 &= (u_2)_x(u_0)_y + (u_1)_x(u_1)_y + (u_0)_x(u_2)_y \\ &\vdots \end{aligned}$$

From the relationship in (15), we obtain

$$\begin{aligned} u_0(x, y, t) &= G(x, t) = e^{xy} + te^{xy} \\ u_1(x, y, t) &= K^{-1} \left[ v^2K \left[ \frac{\partial^2}{\partial x \partial y} A_0(u) - \frac{\partial^2}{\partial x \partial y} (xyB_0(u)) - u_0(x, y, t) \right] \right] \\ &= \frac{-t^2e^{xy}}{2} - \frac{t^3e^{xy}}{6} \\ u_2(x, y, t) &= K^{-1} \left[ v^2K \left[ \frac{\partial^2}{\partial x \partial y} A_1(u) - \frac{\partial^2}{\partial x \partial y} (xyB_1(u)) - u_1(x, y, t) \right] \right] \\ &= \frac{t^4e^{xy}}{24} + \frac{t^5e^{xy}}{120} \\ &\vdots \end{aligned}$$

Which in closed form gives exact solution

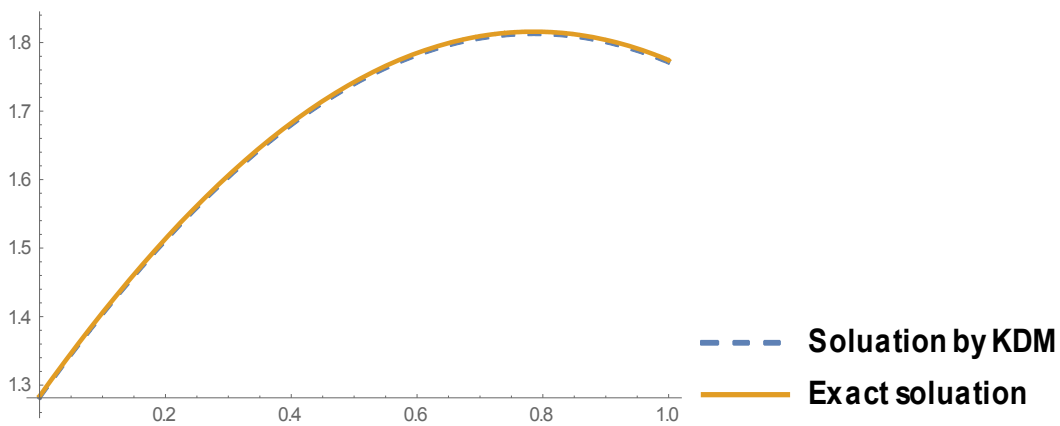
$$u(x, y, t) = \sum_{n=0}^{\infty} u_n(x, y, t) = e^{xy} \left( 1 + t - \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!} + \frac{t^5}{5!} + \dots \right) \quad (22)$$

Thus ,

$$u(x, y, t) = e^{xy}(\sin t + \cos t) \tag{23}$$

**Table 1. Comparison of the absolute errors for KDM results and the exact solution for Example 1 when n=3**

t/x ,y	0.1	0.3	0.5	0.7
0.1	1,42264 E-9	1,54112 E-9	1,80852 E-9	2,29908 E-9
0.3	1,06481 E-6	1,15249 E-6	1,35364 E-6	1,72081 E-6
0.5	2,33822 E-5	2, 53296 E-5	2, 97246 E-5	3, 77873 E-5
0.7	1,80000 E-4	1,94991 E-4	2,28825 E-4	2,90893 E-4
0.9	8,29627 E-4	8,98724 E-4	1,06466 E-3	1,34074 E-4



**Fig. 1. The behavior of the exact solution and KADM solution of U(x;y; t) in case x = y = 0.5; t ∈ [0;1]**

As shown by Fig. 1, comparison of the obtained results with those of the exact solution, reveals that KADM leads to accurate solutions for example 1.

**Example 4.2:**

Let's consider the nonlinear partial differential equation

$$u_{tt} = u^2 \frac{\partial^2}{\partial x^2} (u_x u_{xx} u_{xxx}) - u_x^2 \frac{\partial^2}{\partial x^2} (u_{xx}^3) - 18u^5 + u, 0 < x < 1, t > 0 \tag{24}$$

With the initial condition;

$$u(x, 0) = e^x, u_t(x, 0) = e^x \tag{25}$$

Applying Kamal transform of both sides of Eq. (25) , we obtain,

$$K[u_{tt}(x, t)] = K \left[ u^2 \frac{\partial^2}{\partial x^2} (u_x u_{xx} u_{xxx}) - u_x^2 \frac{\partial^2}{\partial x^2} (u_{xx}^3) - 18u^5 + u \right] \tag{26}$$

Using the differential property of Kamal transform . Eq. (26) can be written as:

$$K[u(x, t)] = ve^x + v^2 e^x + v^2 K \left[ u^2 \frac{\partial^2}{\partial x^2} (u_x u_{xx} u_{xxx}) - u_x^2 \frac{\partial^2}{\partial x^2} (u_{xx}^3) - 18u^5 + u \right] \tag{27}$$

The inverse of Kamal transform implies that:

$$u(x, t) = e^x + te^x + K^{-1} \left[ v^2 K \left[ u^2 \frac{\partial^2}{\partial x^2} (u_x u_{xx} u_{xxx}) - u_x^2 \frac{\partial^2}{\partial x^2} (u_{xx}^3) - 18u^5 + u \right] \right] \quad (28)$$

Following the technique, if we assume an infinite series solution of the form Eqs.(11) and (12), we obtain

$$\sum_{n=0}^{\infty} u_n(x, t) = e^x + te^x + K^{-1} \left[ v^2 K \left[ \sum_{n=0}^{\infty} A_n(u) - \sum_{n=0}^{\infty} B_n(u) - 18 \sum_{n=0}^{\infty} C_n(u) + \sum_{n=0}^{\infty} u_n(x, t) \right] \right] \quad (29)$$

Where  $A_n(u)$ ,  $B_n(u)$  and  $C_n(u)$  are Adomian polynomials that represent nonlinear term. So Adomian polynomials are given as follows:

$$A_n(u) = u^2 \frac{\partial^2}{\partial x^2} (u_x u_{xx} u_{xxx}), \quad B_n(u) = u_x^2 \frac{\partial^2}{\partial x^2} (u_{xx}^3) \text{ and } C_n(u) = u^5$$

Using (13). The few components of the Adomian polynomials are given as follows:

$$\begin{aligned} A_0 &= u_0^2 \frac{\partial^2}{\partial x^2} (u_{0,x} u_{0,xx} u_{0,xxx}) \\ A_1 &= u_1 u_0 \frac{\partial^2}{\partial x^2} (u_{0,x} u_{0,xx} u_{0,xxx}) + u_0^2 \frac{\partial^2}{\partial x^2} (u_{1,x} u_{0,xx} u_{0,xxx}) + (u_{0,x} u_{1,xx} u_{0,xxx}) + (u_{0,x} u_{0,xx} u_{1,xxx}) \\ B_0 &= (u_0)_x^2 \frac{\partial^2}{\partial x^2} ((u_0)_{xx}^3) \\ B_1 &= 2(u_0)_x (u_1)_x \frac{\partial^2}{\partial x^2} ((u_0)_{xx}^3) + (u_0)_x^2 \frac{\partial^2}{\partial x^2} (3(u_0)_{xx}^2 (u_1)_{xx}) \\ C_0 &= (u_0)^5 \\ C_1 &= 5(u_0)^4 u_1 \end{aligned}$$

From the relationship in (15), we obtain

$$\begin{aligned} u_0(x, t) &= e^x + te^x \\ u_1(x, t) &= K^{-1} \left[ v^2 K \left[ A_0(u) - B_0(u) - 18C_0(u) + u_0(x, t) \right] \right] \\ &= \frac{t^2 e^x}{2} + \frac{t^3 e^x}{6} \\ u_2(x, t) &= K^{-1} \left[ v^2 K \left[ A_1(u) - B_1(u) - 18C_1(u) + u_1(x, t) \right] \right] \\ &= \frac{t^4 e^x}{24} + \frac{t^5 e^x}{120} \\ &\vdots \end{aligned}$$

Which in closed form gives exact solution

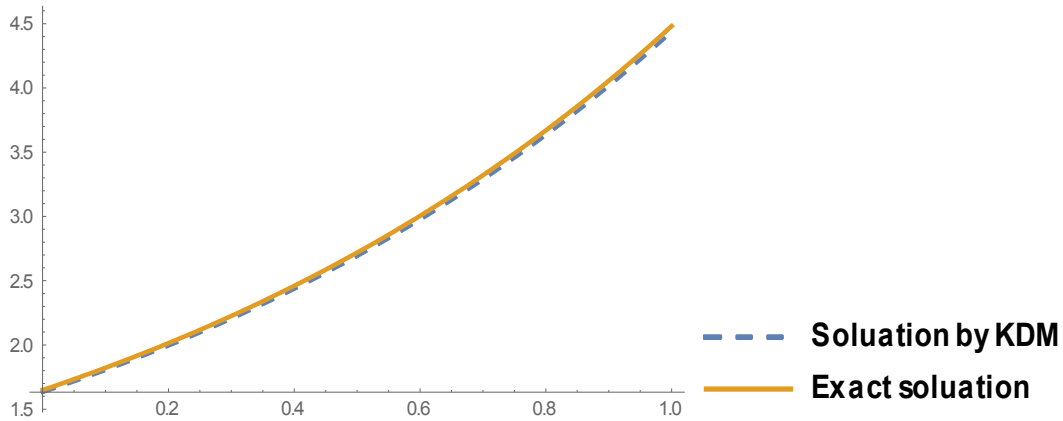
$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t) = e^x \left( 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \frac{t^5}{5!} + \dots \right) \quad (30)$$

Thus,

$$u(x, t) = e^{x+t} \quad (31)$$

**Table 2. Comparison of the absolute errors for KDM results and the exact solution for Example 2 when n=3**

t/x	0.1	0.3	0.5	0.7
0.1	1.55716 E-9	1.90193 E-9	2.32302 E-9	2.83734 E-9
0.2	1.16880 E-6	1.42758 E-6	1.74365E-6	2.12070 E-6
0.3	3. 58102 E-5	3. 15246 E-5	3. 85043 E-5	4.70292 E-5
0.4	2.00357 E-4	2.44717 E-4	2.98898 E-4	3.65075 E-4
0.5	1.33716 E-4	1.14044 E-3	1.39294 E-3	1.70134 E-3



**Fig. 2.** The behavior of the exact solution and KADM solution of  $U(x;y; t)$  of example 2 in case  $x = y = 0;5; t \in [0;1]$

As shown by Fig. 2, comparison of the obtained results with those of the exact solution, reveals that KADM leads to accurate solutions for example 2.

**Example 4.3:**

Let's consider the nonlinear partial differential equation

$$u_{tt} = x^2 \frac{\partial}{\partial x} (u_x u_{xx}) - x^2 (u_{xx}^2) - u, 0 \leq x \leq 1, t > 0 \tag{32}$$

With the initial condition;

$$u(x, 0) = 0, u_t(x, 0) = x^2 \tag{33}$$

Applying Kamal transform of both sides of Eq. (32), we obtain,

$$K[u_{tt}(x, t)] = K \left[ x^2 \frac{\partial}{\partial x} (u_x u_{xx}) - x^2 (u_{xx}^2) - u \right] \tag{34}$$

Using the differential property of Kamal transform . Eq. (34) can be written as:

$$K[u(x, t)] = v^2 x^2 + v^2 K \left[ x^2 \frac{\partial}{\partial x} (u_x u_{xx}) - x^2 (u_{xx}^2) - u \right] \tag{35}$$

The inverse of Kamal transform implies that:

$$u(x, t) = tx^2 + K^{-1} \left[ v^2 K \left[ x^2 \frac{\partial}{\partial x} (u_x u_{xx}) - x^2 (u_{xx}^2) - u \right] \right] \tag{36}$$

Following the technique, if we assume an infinite series solution of the form Eqs.(11) and (12), we obtain

$$\sum_{n=0}^{\infty} u_n(x, t) = tx^2 + K^{-1} \left[ v^2 K \left[ x^2 \frac{\partial}{\partial x} \sum_{n=0}^{\infty} A_n(u) - x^2 \sum_{n=0}^{\infty} B_n(u) - \sum_{n=0}^{\infty} u_n(x, t) \right] \right] \tag{37}$$

Where  $A_n(u)$  and  $B_n(u)$  are Adomian polynomials that represent nonlinear term. So Adomian polynomials are given as follows:

$$A_n(u) = (u_x u_{xx}), \quad B_n(u) = (u_{xx}^2)$$



Using (13). The few components of the Adomian polynomials are given as follows:

$$\begin{aligned}
 A_0 &= (u_{0_x})(u_{0_{xx}}) \\
 A_1 &= (u_{0_x})(u_{1_{xx}}) + (u_{1_x})(u_{0_{xx}}) \\
 A_2 &= (u_{0_x})(u_{2_{xx}}) + (u_{1_x})(u_{1_{xx}}) + (u_{2_x})(u_{0_{xx}}) \\
 &\vdots \\
 B_0 &= (u_0)_{xx}^2 \\
 B_1 &= 2(u_0)_{xx}(u_1)_{xx} \\
 B_2 &= (u_1)_{xx}^2 + 2(u_0)_{xx}(u_2)_{xx} \\
 &\vdots
 \end{aligned}$$

From the relationship in (15), we obtain

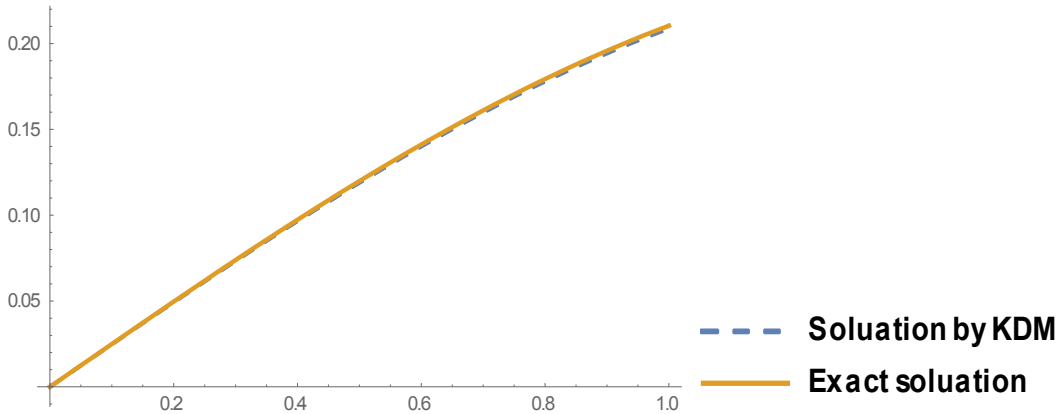
$$\begin{aligned}
 u_0(x, t) &= tx^2 \\
 u_1(x, t) &= K^{-1} \left[ v^2 K \left[ x^2 \frac{\partial}{\partial x} A_0(u) - x^2 B_0(u) - u_0(x, t) \right] \right] = -\frac{t^3 x^2}{6} \\
 u_2(x, t) &= K^{-1} \left[ v^2 K \left[ x^2 \frac{\partial}{\partial x} A_1(u) - x^2 B_1(u) - u_1(x, t) \right] \right] = \frac{t^5 x^2}{120} \\
 u_3(x, t) &= K^{-1} \left[ v^2 K \left[ x^2 \frac{\partial}{\partial x} A_2(u) - x^2 B_2(u) - u_2(x, t) \right] \right] = -\frac{t^7 x^2}{5040} \\
 &\vdots
 \end{aligned}$$

Which in closed form gives exact solution

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t) = x^2 \left( 1 - \frac{t^3}{3!} - \frac{t^5}{5!} - \frac{t^7}{7!} - \dots \right) \tag{38}$$

Thus,

$$u(x, t) = x^2 \sin t \tag{39}$$



**Fig. 3.** The behavior of the exact solution and KADM solution of  $U(x;y; t)$  of example 3 in case  $x = y = 0.5; t \in [0;1]$

As shown by Fig. 3, comparison of the obtained results with those of the exact solution, reveals that KADM leads to accurate solutions for example 3.

**Table 3. Comparison of the absolute errors for KDM results and the exact solution for Example 3 when  $n=3$** 

$t/x$	<b>0.1</b>	<b>0.3</b>	<b>0.5</b>	<b>0.7</b>
0.1	0	0	0	0
0.3	0	0	0	0
0.5	196024 E-16	176248 E-15	4. 89886-15	9.60343 E-15
0.7	1.55232 E-14	1.39708 E-13	3.88078 E-13	7.60614 E-13
0.9	4.06630 E-13	3.65968 E-12	1.01658 E-11	1.99248 E-11

## 5 Conclusion

In the present paper, Kamal decomposition transform (KADT) was applied for solving non-linear wave – like equations with variable coefficient . The method was illustrated by three different examples , and their solutions were compared by that given by the exact solution , the results reveal that Kamal decomposition method for solving non-linear wave like equation is very efficient, simple and can be applied to other non-linear problems.

## Competing Interests

Author has declared that no competing interests exist.

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