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# Variation Mass Effects on the Stability of Libration Points of Restricted Three Body Problem (Laplace Transformation)

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***Authors' contributions***

*This work was carried out in collaboration between all authors. Author MNI designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors SHY and FME managed the analyses of the study. Author MNI managed the literature searches. All authors read and approved the final manuscript.*

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## Abstract

In this work a dynamical system of three bodies with decreasing mass with respect to time is deduced. The Jean's law is applied onto equations of motion. The equations of motion in this case are differed from that with constant mass only by small perturbing force considered as the variation of mass. This system is solved by using Laplace and Inverse Laplace Transformation. An application is done on the Earth–Moon-|Spacecraft system. The results obtained are compared with the previous work obtained by Explicit Rung-Kutta method and it was in a good agreement.

*Keywords: Laplace and Inverse Laplace transformation; dynamical systems; restricted three body problem (RTBP); perturbation methods.*

## 1 Introduction

Jean [1] has considered the two-body problem with variable mass in the studying the evaluation of binary system, this enabled many important results from the physical and mathematical points. Omarov [2] has

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considered the restricted two body problem of perturbed motion with variable mass and has shown that permissible rates of mass variation of binary system exerts a small perturbation on the motion of the binary.

Singh [3] has studied linear and nonlinear stability of the libration points in the RTBP with variable mass. Then Singh and Leke [4] has studied the stability of the photo-gravitational effect on RTBP with the variable mass of the primaries. They have found their exist stability about the triangular libration points for $0 < \mu < \mu_c$ and not stable for $\mu_c < \mu < \frac{1}{2}$, and they observed that the collinear libration points are unstable. Zangh et al. [5] have studied the triangular libration points under the effect of photo-gravitational with the variable mass, their results provided that the motion around these points are unstable for the problem with constant mass, and evolved into the problem with decreasing mass. Recently Abouelmaged and Mostafa [6] have studied the dynamics of the third body with change in mass proposed by Jean's law. They have obtained the locations of libration points, and they have found this points lie out-of-plane, in special case of a non-isotopic variable of mass.

In this work we use the Laplace Transformation who's developed by the French mathematician Pierre-Simon Laplace, this method is powerful tool for solving certain types of initial value problem. The strategy is to convert integral, differential and integral-differential equations into algebraic equations where solutions can be easily obtained. Then the Inverse Laplace Transform, can be applied to retrieve the solutions of the original problems. This makes the problem much easier to be understood, analyzed and solved. Laplace Transform are particularly effective on differential equations with forcing functions that are piecewise defined, periodic or impulsive. From this method we can find quantities solutions for the system of non-linear differential equations and we can study the stability about the libration points [7].

***Definition***. The Laplace transform of a function, f(t), for $t \geq 0$ is defined by

$$\int_0^\infty e^{-st} f(t) dt \qquad (1)$$

Where

- The improper integral must convergence (i.e. the limit exists and is finite) for at least one value of s.
- t is real and called the time variable.
- s is complex and called the frequency variable.
- The resulting expression is a function of s, which symbolled by F(s).

***Definition***. The Inverse Laplace transform of the function F(s) is given by

$$\frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} F(s) e^{st} ds \qquad (2)$$

Where the integral is taken over a line in the region of convergence and $\sigma$ is large enough that F(s) is defined for real $s \geq \sigma$.

This formula (integration in complex plane) is very difficult to be applied directly, so we will use a different approach.

Lagrange has proposed a method that had never been used until then, that of considering only the distances between the three bodies rather than their absolute positions [8]. Through this method, he has found that there are exactly five different configurations, which give the probabilities of motion of the third body, and are called the five Lagrange points or five libration points [9].

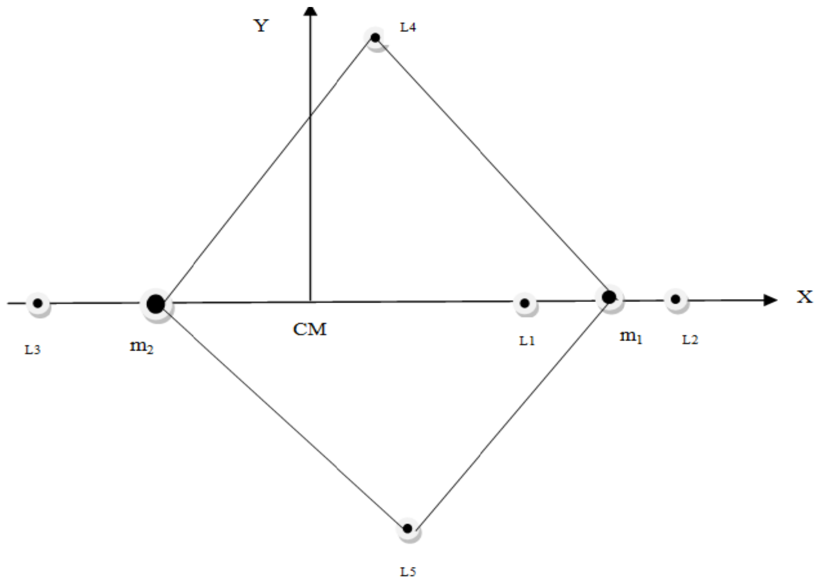


Fig.1: Liberation points in 3-body problem

In our work we defined the differential equations of motion of the RTBP. We supposed that the mass of the finite bodies is constants and the mass of the satellite decreases with respect to the time. The problem is restricted in the sense that the mass of satellite is taken to be infinitesimally small, so the force of attraction on the two primaries by the satellite is neglected [10].

- Jean's law enabled to form the equations of motion. Which can be solved by Laplace Transformation, which enabled us to find an analytical solution for the nonlinear differential equations.

## 2 Equations of Motion

Let us suppose that the masses of the primaries bodies $m_1$ and $m_2$ are constant and the mass of the third body (satellite) m is varied with respect to time t.

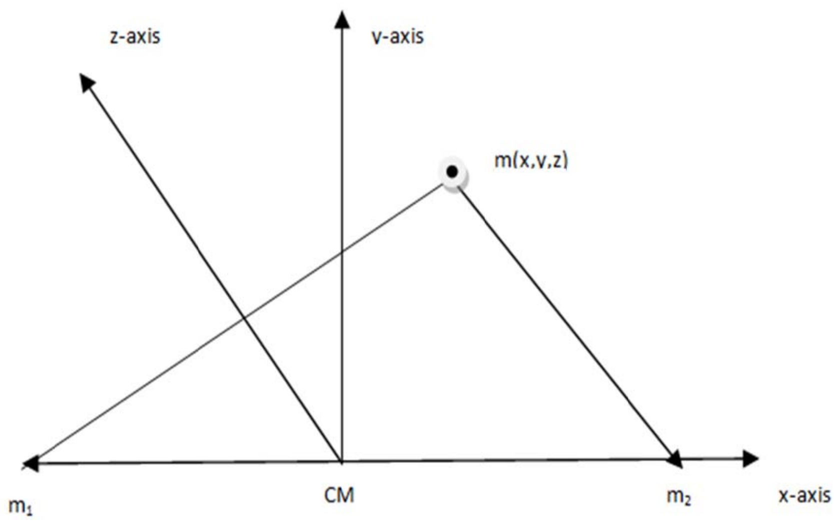


**Fig. 2. The geometric of the problem**

Now, let us consider a coordinate system XYZ, with origin at CM (the center of mass of the two primaries) and the frame rotating relative to the initial space with angular velocity $\omega$ about Z axis. Without loss of generality we can choose the coordinates system such that X-axis lies along the line joining the primaries masses $m_1$ and $m_2$ [10].

Let

$$G(m_1+m_2)=1;\ \mu=\frac{m_2}{m_1+m_2};\ \ Gm_1=1-\mu;\qquad Gm_2=\mu \tag{3}$$

Let the distance between $m_1$, $m_2$ be unit distance, and the position of the satellite of mass m is given by the coordinates (X, Y, Z). Let the radius vector from m to $m_1$ and from m to $m_2$ be $r_1$ and $r_2$ respectively. The kinetic energy in the rotation frame of reference XYZ is given by

$$\begin{aligned}T&=\frac{1}{2}\,m[(\dot{x}-\ \omega)+(\dot{y}+\omega x)^2+\dot{z}^2]\\&=\frac{1}{2}\,m[(\dot{x}^2+\dot{y}^2+\dot{z}^2)+\omega(x\dot{y}-\dot{x}y)+\omega^2(x^2+y^2)\,]\\&=T_2\ +\ T_1\ +T_0\end{aligned} \tag{4}$$

Where

$$T_2=\frac{1}{2}\,m(\dot{x}^2+\dot{y}^2+\dot{z}^2);\ \ T_1=\frac{1}{2}\,m\ \ \omega(x\dot{y}-\dot{x}y);\ T_0=\frac{1}{2}\,m\omega^2(x^2+y^2) \tag{5}$$

The potential energy is given by

$$V\ \ =-Gm\left(\frac{m_1}{r_1}+\frac{m_2}{r_2}\right) \tag{6}$$

Where G is the gravitational constant

$${r_1}^2=(x-1+\mu)^2+y^2+z^2;\ \ \ {r_2}^2=(x-\mu)^2+\ y^2+z^2 \tag{7}$$

$m_1(1-\mu,0,\ 0)\ \ and\ m_2(\mu,0,0)$ are coordinates of the finite bodies. We know that Lagrangian

$$L=T-V \tag{8}$$

Now, let us introduce the modified potential energy

$$U=V-T_0 \tag{9}$$

$$U=-\left[\frac{(1-\mu)}{r_1}+\frac{\mu}{r_2}+\frac{1}{2}\,m\omega^2(x^2+y^2)\right] \tag{10}$$

Then Lagrangian can be written in the form

$$L=\frac{1}{2}\,m[(\dot{x}^2+\dot{y}^2+\dot{z}^2)+\omega(x\dot{y}-\ x\dot{y})-\ U] \tag{11}$$

The equations of motion can be written in the form

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{x}}\right)-\frac{\partial L}{\partial x}=0 \tag{12.1}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{y}}\right)-\frac{\partial L}{\partial y}=0 \tag{12.2}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) - \frac{\partial L}{\partial z} = 0 \tag{12.3}$$

From Eq. (12.1), Eq. (12.2) and Eq. (12.3) we obtained,

$$\ddot{x} + \frac{\dot{m}}{m}[\dot{x} - \omega y] - 2\omega\dot{y} = -\frac{1}{m}u_x \tag{13.1}$$

$$\ddot{y} + \frac{\dot{m}}{m}[\dot{y} + \omega x] - 2\omega\dot{x} = -\frac{1}{m}u_y \tag{13.2}$$

$$\ddot{z} + \frac{\dot{m}}{m} z = -\frac{1}{m}u_z \tag{13.3}$$

Where

$$u_x = -aMw^2 + \frac{\mu(-1+a+\mu)}{(c^2+f^2+(-1+a+\mu)^2)^{3/2}} + \frac{(1-\mu)(a+\mu)}{(c^2+f^2+(a+\mu)^2)^{3/2}} \tag{14}$$

$$u_y = -cMw^2 + \frac{c\mu}{(c^2+f^2+(-1+a+\mu)^2)^{3/2}} + \frac{c(1-\mu)}{(c^2+f^2+(a+\mu)^2)^{3/2}} \tag{15}$$

$$u_z = \frac{f\mu}{(c^2+f^2+(-1+a+\mu)^2)^{3/2}} + \frac{f(1-\mu)}{(c^2+f^2+(a+\mu)^2)^{3/2}} \tag{16}$$

Now from Jean's law

$$\frac{dm}{dt} = -\alpha\, m^n \tag{17}$$

Then

$$\frac{\dot{m}}{m} = \frac{-\alpha m^n}{m} = -\alpha m^{n-1} = -\beta \tag{18}$$

Where $\alpha$ is constant coefficient and the value of exponent n is lies in $0.4 \le n \le 4.4$ [3].

$$\ddot{x} + \beta[\dot{x} - \omega y] - 2\omega\dot{y} = -\frac{1}{m}u_x \tag{19.1}$$

$$\ddot{y} + \beta[\dot{y} + \omega x] - 2\omega\dot{x} = -\frac{1}{m}u_y \tag{19.2}$$

$$\ddot{z} + \beta z = -\frac{1}{m}u_z \tag{19.3}$$

the above nonlinear system (19) can be solved by using Laplace Transformation. Beginning with Eq, (19.1) applying Laplace Transformation, then

$$L\{\ddot{X}\} + \beta L\{(\dot{x} - \omega y)\} - 2\omega L\{\dot{y}\} = \frac{-1}{m}L\{u_x\}$$
$$S^2X(S) - Sx(0) - \dot{x}(0) + \beta[SX(S) - x(0) - \omega Y(S)] - 2\omega[SY(S) - y(0)] = \frac{-1}{mS}U_X$$
$$[S^2 + \beta S]X(s) - [2\omega S + \beta\omega]Y(S) - aS + [2\omega c - a\beta - b] = \frac{-U_x}{mS} \tag{20.1}$$

Where $x(0) = a$ ; $\dot{x}(0) = b$; $y(0) = c$; $\dot{y}(0) = d$

Similarly, take the Laplace Transform for Eqs. (19.2), and (19.3) yields,

$$\omega[2S + \beta]X(S) + [S^2 + \beta S]Y(S) - cS - [c\beta + 2a\omega + d] = \frac{-U_x}{mS} \tag{20.2}$$

$$[S^2+\beta S]\,Z(S)-e[S+1]-f=\frac{-U_z}{mS} \tag{20.3}$$

Where $e=z(0)\,;\ f=\dot{z}\,(0)$

From Eqs. (20.1) , (20.2), and (20.3) after some little algebraic reductions Y(S), X(S) ,and Z(S) are obtained as,

$$Y(S)=\frac{mS\{(c(S^2+\beta S)-a(\beta\omega+2\omega S))S+(\beta\omega+2\omega S)c_1+(S^2+\beta S)c_2\}+\{(\beta\omega+2\omega S)U_x-(S^2+\beta S)U_y\}}{mS\{(s^2+\beta S)^2+(\beta\omega+2\omega S)^2\}} \tag{21}$$

$$X(S)=\frac{mS\{(a(S^2+\beta S)+c(\beta\omega+2\omega S))S-\{(S^2+\beta S)c_1-(\beta\omega+2\omega S)c_2\}-\{(S^2+\beta S)U_x+(\beta\omega+2\omega S)U_y\}}{mS\{(s^2+\beta S)^2+(\beta\omega+2\omega S)^2\}} \tag{22}$$

$$Z(S)=\frac{mS[e(S+1)+f]-U_z}{mS(S^2+\beta S)} \tag{23}$$

Where $c_1=(2\omega-a\beta-b);\ c_2=(c\beta+2\omega a+d)$ (24)

Take the Inverse Laplace Transform we obtained,

$$y(t)=A-(a_1+b_1 i)\big((a_{11}-b_{11}i)\mathrm{Cosh}[Bt]+(a_{11}-b_{11}i)\mathrm{Sinh}[Bt]\big)-(\mathrm{A}_1-A_2 i)(\mathrm{Cos}[Ct]-i\mathrm{Sin}[Ct])\big((a_{12}+b_{12}i)\mathrm{Cosh}[Bt]+1.\mathrm{Cosh}[(B+B_1 i)t]-(a_{12}+b_{12}i)\,\mathrm{Sinh}[Bt]+1.\mathrm{Sinh}[(B+B_1 i)t]\big) \tag{25}$$

$$\mathrm{x(t)}=(\mathrm{A}_2+A_1 i)(\mathrm{Cos}[C_1 t]-i\mathrm{Sin}[C_1 t])\big((a_{12}-b_{12}i)\mathrm{Cosh}[Bt]\,(a_{13}+b_{13}i)\mathrm{Cosh}[(B+Ci)t]-(a_{14}-b_{13}i)\mathrm{Cosh}[(B+C_2 i)t]+\mathrm{Cosh}[(B+b_{14}i)t]+(a_{12}-b_{12}i)\mathrm{Sinh}[Bt]-(a_{13}+b_{13}i)\mathrm{Sinh}[(B+Ci)t]-(a_{14}-b_{13}i)\mathrm{Sinh}[(B+C_2 i)t]+\mathrm{Sinh}[(B+b_{14}i)t]\big) \tag{26}$$

$$z(t)=A_3+A_4 t-A_3\mathrm{Cosh}[b_{12}t]-A_3\mathrm{Sinh}[b_{12}t] \tag{27}$$

Where A, $A_1$, $A_2$, $A_3$, $A_4$, B, $B_1$, C, $C_1$, $C_2$, $a_1$, $b_1$, $a_{11}$, $b_{11}$, $a_{12}$, $b_{12}$, $a_{13}$, $b_{13}$, $a_{14}$, $b_{14}$, are constants evaluated from the initial condition and the locations of the libration points.

# 3 Results and Discussion

## 3.1 Numerical analysis

The Explicit Rung-Kutta method is used to find the numerical solution for the system of the differential equations (19.1), (19.2) and (19.3), a code of Mathematica was constructed to obtain the Libration points and to study their stability. Another code to obtain the Laplace and Inverse- Laplace Transformation (analytical solution). These two codes are applied to the system Earth-Moon-Spacecraft in which the coordinates of the libration points are calculated and the stability conditions are taken into account.

## 3.2 The collinear libration points ($L_1$, $L_3$ and $L_2$)

From the results obtained, it is found that the location of $L_1$ at (0.9, 0). The stability of $L_1$ is obtained analytically and numerically taken into account various values of the variation mass effect ( ). Figs. (3.1.a) and Fig. (3.1.b) illustrate the stability about $L_1$ for about 600 periods in both analytical and numerical methods ( $\beta=-0.1$) . Figs. (3.2.a) and Fig. (3.2.b) illustrates the stability of phase space of third body about $L_1$ for the same . In the same way the Figs.( 3.3.a), Figs( 3.3.b) , Fig.(3.4.a) and Fig.(3.4.b) illustrate the stability for about 400 periods with $\beta=-0.01$. Similarly the Fig. (3.5.a), Fig.( 3.5.b), Fig.(3.6.a) and Fig. (3.6.b) illustrate the stability and phase space of third body about $L_1$ for about 100 period with $\beta=-0.00001$. From the above rustles we conclude that the variation of mass have higher effected on the stability of the spacecraft about the libration point $L_1$, an we found that there exist Inverse proportional relation between the decreasing effected of the variation of mass and the stability of the spacecraft in the periodic orbit,

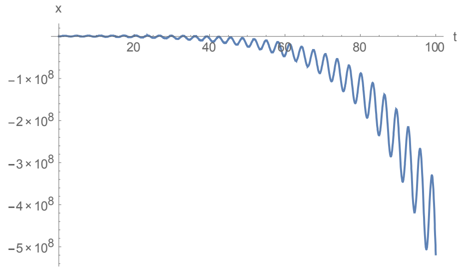


**Fig. 3.1.a. The analytic sol. x(t) about $L_1$, $\beta = -0.1$**

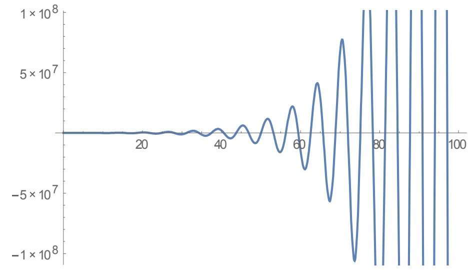


**Fig. 3.1.b. The numerical sol. x(t) about $L_1$, $\beta = -0.1$**

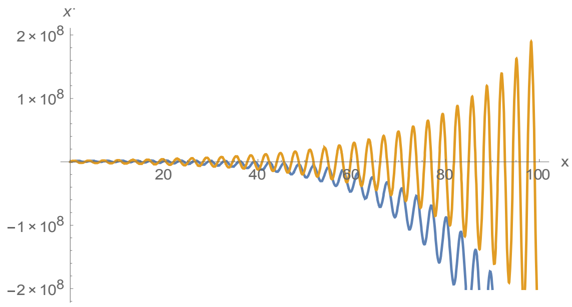


**Fig. 3.2.a. The phase space about $L_1$, $\beta = -0.1$ analytical**

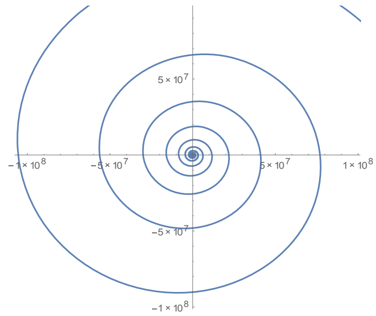

**Fig. 3.2.b. The phase space about $L_1 = \beta = -0.1, numerical$**

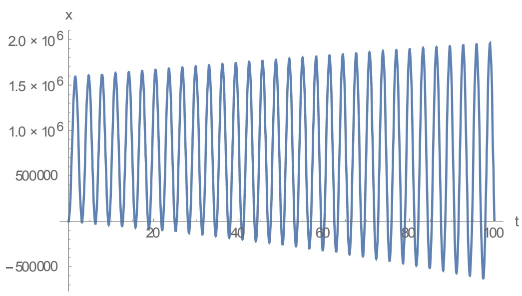


**Fig. 3.3.a. The analytical sol. x(t) about $L_1$, $\beta = -0.01$**

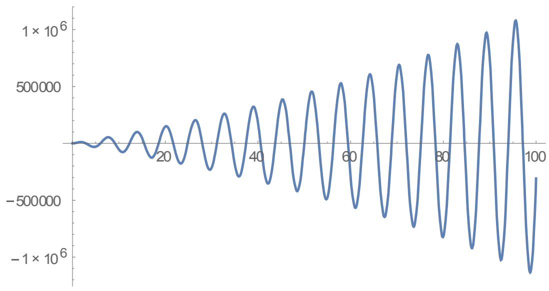


**Fig. 3.3.b. The numerical sol. x(t) about $L_1$, $\beta = -0.01$**

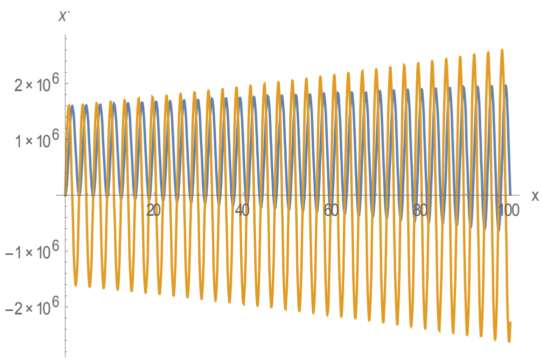


**Fig. 3.4.a. The phase space about $L_1$, $\beta = -0.01$ analytical**

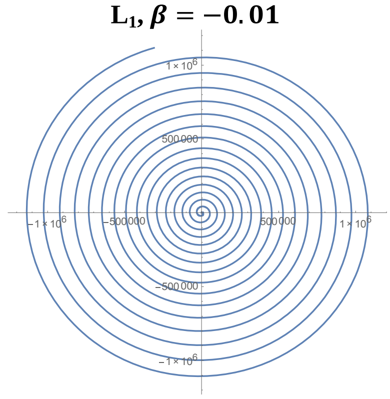

**Fig. 3.4.b. The phase space about $L_1$, $\beta = -0.01$ numerically**

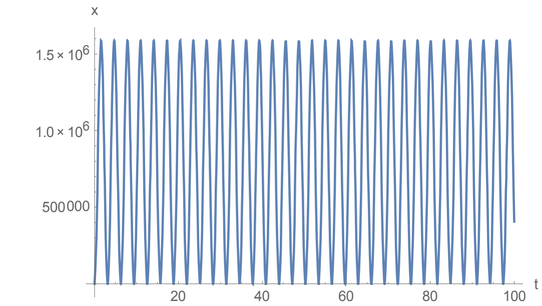


**Fig. 3.5.a. The analytical sol. x(t) about $L_1$ ,$\boldsymbol{\beta} = -0.00001$**

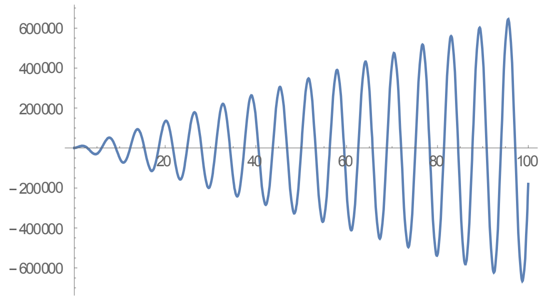


**Fig. 3.5.b. The numerical sol. x(t) about $L_1$, $\boldsymbol{\beta} = 0.00001$**

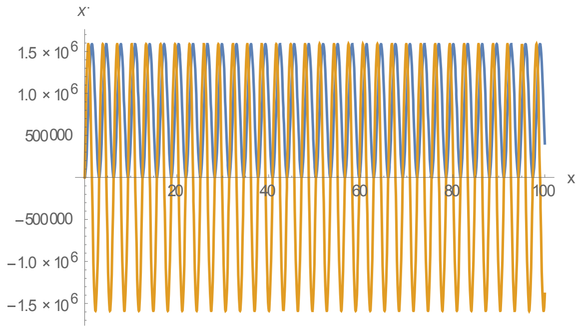


**Fig. 3.6.a. The phase space about $L_1$, $\boldsymbol{\beta} = -0.00001$ analytical**

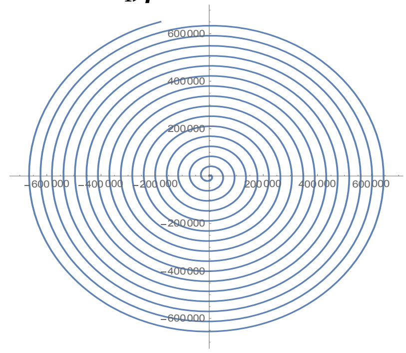


**Fig. 3.6.b. The phase space about $L_1$, $\boldsymbol{\beta} = -0.00001$ numerical**

In the same way, at $L_3$ (- 0.9, 0) Fig. (3.7.a) and Fig. (3.7.b) also at $L_2$ (1.2 ,0) Figs. (3.8.a) and Figs. (3.8.b) have the same convolution from the results obtained as mentioned in the case of $L_1$.

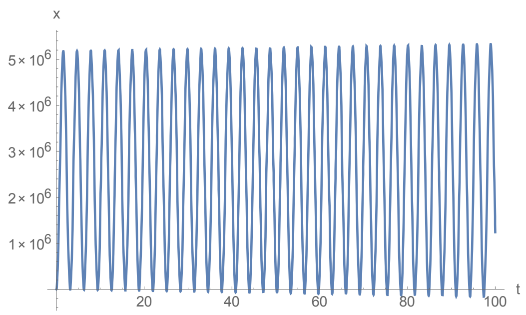


**Fig. 3.7.a. The analytic sol. x(t) about $L_3$, $\boldsymbol{\beta} = -0.1$**

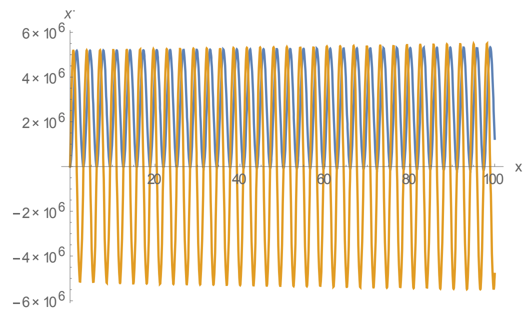


**Fig. 3.7.b. The phase space about $L_3$, $\boldsymbol{\beta} = -0.1$ analytically**

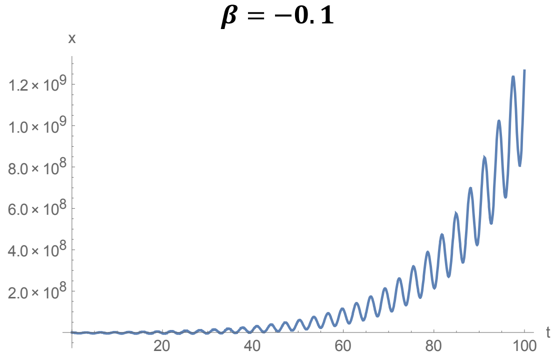


**Fig. 3.8.a. The analytical sol. x(t) about $L_2$, $\boldsymbol{\beta} = -0.1$**

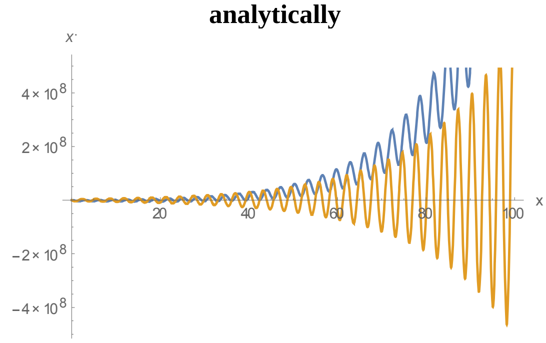


**Fig. 3.8.b. The phase space about $L_2$, $\boldsymbol{\beta} = -0.1$ analytical**

### 3.3 The triangular liberation points ($L_4$ and $L_5$)

From the results obtained that at $L_4\left(0.8\ ,\ \frac{\sqrt{3}}{2}\right)$, Fig. (3.9.a) and Fig. (3.9.b) shows that the variation of the general solution with the time, and it's illustrates that the results obtained analytically and numerically.

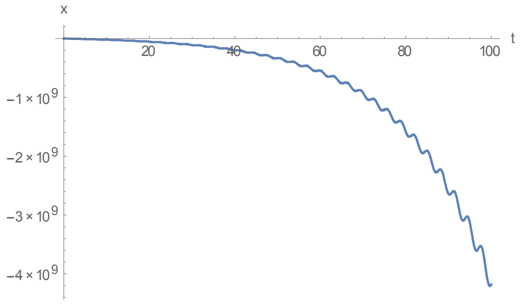


**Fig. 3.9.a. The analytical sol. x(t) about $L_4$, $\beta = -0.1$**

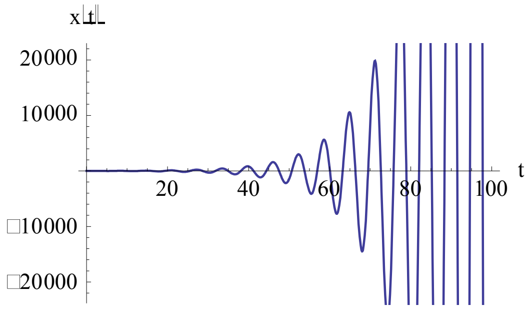


**Fig. 3.9.b. The numerical sol. x(t) about $L_4$, $\beta = -0.1$**

Fig.(3.10.a ) and Fig.(3.10.b) illustrates the phase space about $L_4$ with $\beta = -0.1$

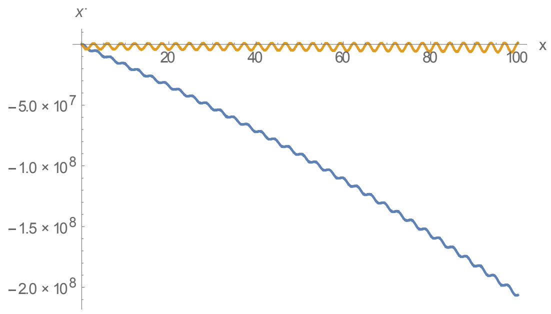


**Fig. 3.10.a. The phase space about $L_4$, $\beta = -0.01$ analytically**

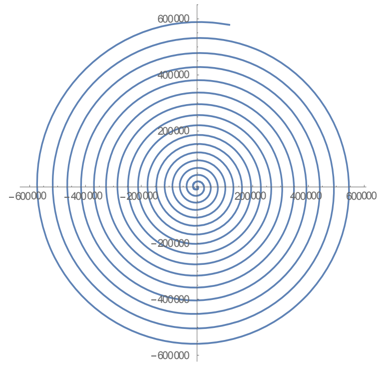


**Fig. 3.10.b. The phase space about $L_4$, $\beta = -0.01$ numerically**

In the same way, at $L_5\left(0.8\ ,-\frac{\sqrt{3}}{2}\right)$ we found a periodicity behavior about this point. Fig. (3.11.a) illustrate the analytically solution obtained with $\beta = -0.1$ and Fig. (3.11.b) illustrate the phase space with $\beta = -0.1$.

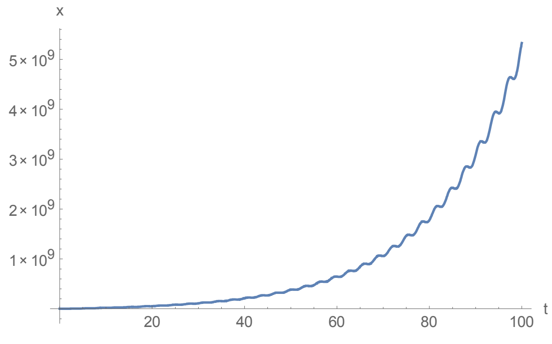


**Fig. 3.11.a. Analytical sol. x(t) about $L_5$, $\beta = -0.1$**

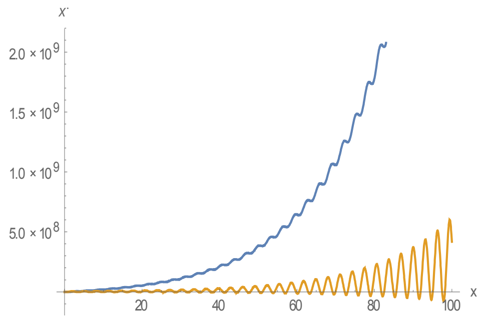


**Fig. 3.11.b. The phase space about $L_5$, $\beta = -0.1$**

## 3.4 Trajectories around the libration points

The trajectories about $L_2$ and $L_4$ are studied for different values of $\beta$, Table [1.1] illustrates the result for $L_2$, Fig. (3.12) and Fig. (3.13) illustrates trajectories about $L_2$.

**Table [1.1]**

| $L_2(1.2,0)$ | $\beta$ | Eccentricity | Periodic time |
|---|---|---|---|
| | .1 | 0.99849 | 125.6637 |
| | .01 | 0.99849 | 125.6637 |
| | .001 | 1 | 125663.7061 |

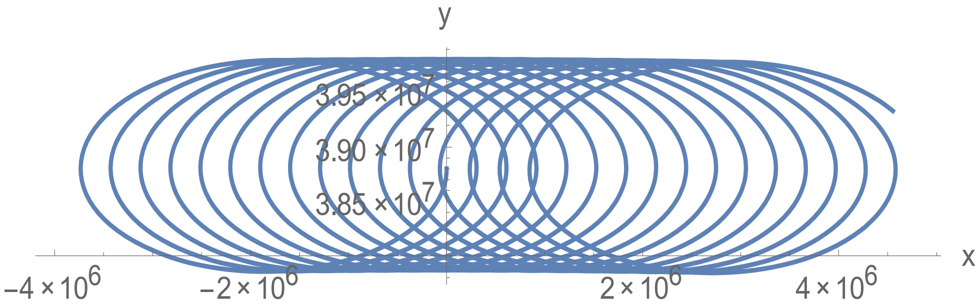


**Fig. 3.12. The trajectory around $L_2(1.2,0)$ at $\beta = .1$**

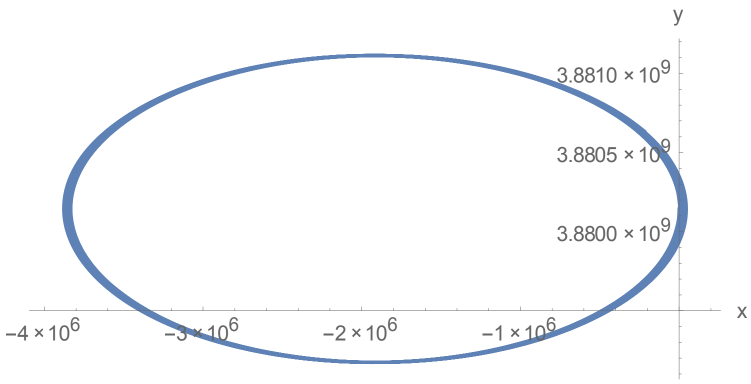


**Fig. 3.13. The trajectory around $L_2(1.2,0)$ at $\beta = .001$**

In the same manner Table [1.2] illustrates the results for $L_4$, Fig. (3.14) and Fig. (3.15) illustrates the trajectories about $L_4$.

**Table [1.2]**

| $L_4(.8, \frac{\sqrt{3}}{2})$ | $\beta$ | Eccentricity | Periodic time |
|---|---|---|---|
| | .1 | 1 | 12566.37 |
| | .01 | 0.40029 | 125.66 |
| | .001 | 0.99597 | 1256.63 |
| | .0001 | 0.99995 | 12566.370 |
| | .00001 | 1 | 1256637.061 |

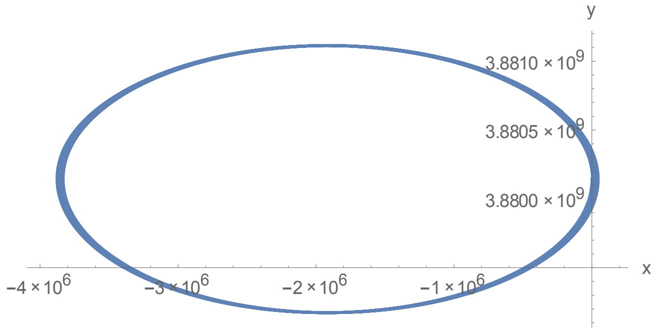


**Fig. 3.14. The trajectory around $L_4(.8, \frac{\sqrt{3}}{2})$ at $\beta = .1$**

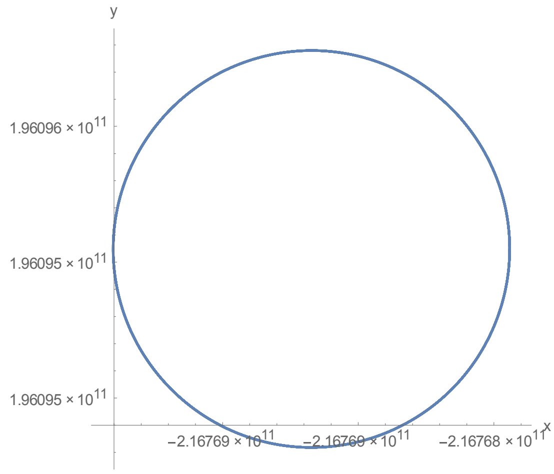


**Fig. 3.15. The trajectory around $L_4(.8, \frac{\sqrt{3}}{2})$ at $\beta = .00001$**

## 3.5 Relation between the variation of mass (β) and the velocity

The velocity is calculated for the different values of the variation of mass (β) at certain time. It's found there exist that the inverse relation between (β) and the velocity as shown in Fig. (3.16) and Fig. (3.17) at $L_2$ and $L_4$. And $V_0$ represent the escape velocity of the spacecraft.

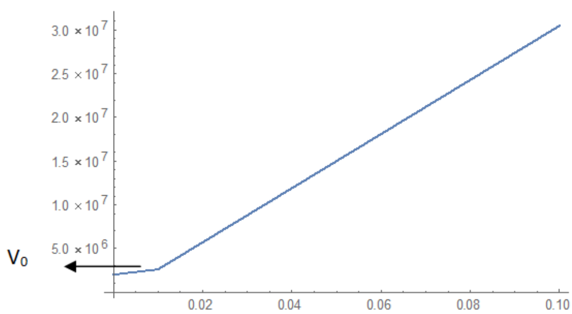


**Fig. 3.16. The relation between V at $\beta = .1, .01 .001, .0001, .00001$ at $L_2$ (1.2 ,0)**

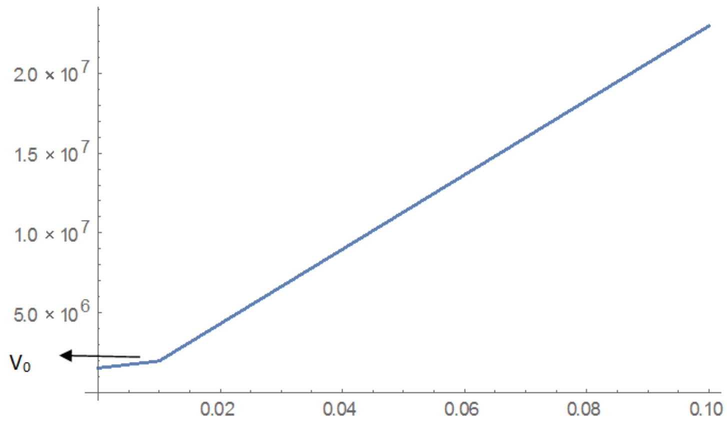


**Fig. 3.17. The relation between V at $\beta = .1, .01.001, .0001, .00001$ at $L_4(.8, \frac{\sqrt{3}}{2})$**

## 4 Conclusions

It is s clear that the Laplace transforms method which has an important property and it is convergence (i.e. limit exists and finite for at least one value) this enables us to find the analytical solution for the system of non-linear ODEs. This help us to study the libration points $L_1$, $L_2$, $L_3$ at the same point and in the neighborhood and we find that, there exist *stability at* the libration points when the decreasing of the mass variation is considered, and we find that strong effect for the mass variation on libration points is existed, this variation take Inverse relationship between the value of mass variation (β ) and the velocity. At the Triangular points ($L_4$, $L_5$) there exists stability for a period of time and disturbance occurs when the value of mass variation increasing, we return this to two possibilities: firstly During the period of disturbance, the effect of the variation mass is strong in this period. Which cause the spacecraft moves from one orbit to another in this period and it is stable and so on again, in space mission this called maneuvers. The second is the variation of mass affect the velocity to change the motion of spacecraft i.e. accelerate this motion.

## Competing Interests

Authors have declared that no competing interests exist.

## References

[1] Jeans JH. Astronomy and Cosmogony (Cambridge). 1928;352.

[2] Omarov TB. Two-body motion with corpuscular radiation. Soviet Astronomy. 1964;7:707.

[3] Singh J. Photogravitational restricted three-body problem with variable mass. Indian Journal of Pure & Applied Mathematics. 2003;34(2):335-341.

[4] Singh J, Leke O. Stability of the photogravitational restricted three-body problem with variable masses. Astrophysics and Space Science. 2010;326(2):305-314.

[5] Zhang MJ, Zhao CY, Xiong YQ. On the triangular libration points in photogravitational restricted three-body problem with variable mass. Astrophysics and Space Science. 2012;337(1):107-113.

[6] Abouelmagd EI, Mostafa A. Out of plane equilibrium points locations and the forbidden movement regions in the restricted three-body problem with variable mass. Astrophysics and Space Science. 2015;357(1):58.

[7] Poularikas AD, (Ed). Transforms and Applications Handbook. CRC Press; 2010.

[8] Lagrange, Joseph-Louis. Essai sur le probleme des trois corps. Prix de l'académie royale des Sciences de Paris. 1772;9:292.

[9] Howell KC. Three-dimensional, periodic, 'halo' orbits. Celestial Mechanics. 1984;32(1):53-71.

[10] Shrivastava AK, Ishwar B. Equations of motion of the restricted problem of three bodies with variable mass. Celestial Mechanics and Dynamical Astronomy. 1983;30(3):323-328.




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